A

NEW and COMPLEAT

TREATISE

OF THE

Doctrine of *Fractions*,

VULGAR and DECIMAL;

Containing not only all that hath hitherto been publish'd on this SUBJECT; but also many other compendious Usages and Applications of them, never before extant. Together with a Compleat Management of CIRCULATING NUMBERS, which is entirely New, and absolutely necessary to the right using of FRACTIONS.

To which is added, an

Epitome of DUODECIMALS,

AND AN

Idea of MEASURING.

The whole is adapted to the meanest Capacity. and very useful to BOOK-KEEPERS, GAUGERS, SURVEYORS, and to all Persons whose Business requires Skill in ARITHMETICK.

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THE PREFACE.

SINCE the abſolute Neceſſity of knowing Fractions, and the extreme Uſefulneſs of them in Practice is ſo well known, I ſhall be ſilent herein; and only ſhew in what manner I have laid down the Doctrine of them, and wherein I differ from others in the practical Management.

In the firſt Seven Chapters, I have handled general, or vulgar Fractions, and in the following Chapters particular, as Decimal and Duodecimal Fractions.

In order to the ready Management of vulgar Fractions, I have first laid down the Methods and Rules commonly given, and afterwards added many others, illustrated by Examples, which will greatly shorten the Work, and always give the Result in its least Terms; which is an Advantage very well worth Pursuit.

And tho' the 3d, 4th, 5th, 6th, and 7th Sections of the 1st Chapter may seem foreign to the Business of this Treatise, whoever considers them thoroughly, will find them of Use, in the ready Management of vulgar Fractions.

In the 8th, 9th, 10th, 11th, 12th, 13th, 14th and 15th Chapters, after the handling of compleat Decimals, I have shewn how to use without Loss, such as do Circulate; and also how far we may trust to Results arising from Approximates.

If Decimals are at any time used without the like Management, many gross Errors must arise, as will be manifest to every one that considers Conclusions from Approximates.

For he that only knows how to express Money, Weight, &c. Decimally, will find, that such Expressions much oftener circulate, than at any place terminate. And consequently,

quently, if he works ſuch Decimals as Approximates, his Reſults will have ſo many uncertain places, eſpecially in future Operations, *which muſt, as well as hitherto it hath diſhearten'd the Practicers of Decimals.*

Tho' this Method of uſing Fractions is abſolutely neceſſary, yet no Treatiſe hitherto extant hath ſufficiently handled it; for the Reverend Dr. Wallis, *in the 89th Chapter of his Hiſtory of* Algebra, *publiſh'd in the Year* 1685, *treating of the Laws of Circulating Numbers, concludes thus,* I have inſiſted the more particularly on this, becauſe I don't remember I found it ſo conſider'd by any other. *And ſince him, Mr.* Jones *in his* Synopſis Matheſeos, *recites conciſely part of what the Doctor had explained before at large. And laſtly, Mr.* Ward, *in his* Young Mathematician's Guide, *contents himſelf with informing his Reader,* That Numbers will circulate. *But neither of theſe ſo much as hint at the Methods of uſing them.*

In the 3d Section *of the 8th* Chapter, *I have laid down all the Laws of Circulations hitherto publiſh'd, and many more of my own; but have omitted their Demonſtrations (which at firſt I deſign'd to annex) becauſe they depend on Propoſitions in the 7th, 8th and 9th Books of* Euclid; *which by our modern*

modern Masters are, for I know not what Reasons, termed useless Curiosities; and so by their Scholars not look'd into; and unheard of by such as are Arithmeticians only.

But the Reverend Mr. Brown, *in his System of* Decimal Arithmetick, *manages such interminate Decimals as have a single Digit continually repeated; but in Multiplication useth only such Factors as will produce a single Repetend in the Product (being, as I suppose, unwilling so much as to mention compound Repetitions) and in Division leaves the Practitioner towork without Exactness.*

These are all the Authors I have met with, that have so much as once written of Circulations. Yet in Gratitude I cannot forget my Friend Mr. Robert Favell *School-Master in St.* Giles's *in the Fields, whose Hints and particular Methods have contributed to my Discovery of the Nature and Laws of Circulating Figures.*

The Approximation of Proportions is from the 10*th and* 11*th Chapters of Dr.* Wallis's History of Algebra.

The Epitome of Duodecimals is all I ever found necessary in our Customary Measuring, wherein only they are used.

The

The whole is laid down in ſo eaſie a Manner, as to be intelligible to any Reader who knows Multiplication, Diviſion, *and* Extraction of Roots in Integers, *and* Reduction of Money, Weights *and* Meaſures, &c.

To conclude, that this Treatiſe, as being ſo much more uſeful than any other Book of this Subject yet extant, may be kindly accepted, is the Deſire of

SAMUEL CUNN.

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A TREATISE OF FRACTIONS.

CHAP. I.

Of *PRIMES* and *MEASURES*.

SECT. I.

DEFINITIONS.

1. UNITY is that by which every thing that is, is called One.

2. Number is a Multitude compos'd of Units.

3. A leſſer Number is an aliquot Part of a greater, when the leſſer exactly meaſures the greater; and the greater is then called a Multiple of the leſſer.

4. A leſſer Number is an aliquant Part of a greater, when the leſſer doth not exactly meaſure the greater.

So 4 is an aliquot Part of 12, and 12 a Multiple of 4, becauſe 3 times 4 is 12; but 5 is an aliquant Part of 12, becauſe 2 times 5 is 10, and 3 times 5 is 15, and conſequently 5 doth not exactly meaſure 12.

5. A Prime Number is that which is meaſured only by an Unit.

6. A Compoſed Number is that which ſome Number meaſureth.

7. Numbers prime to one another, are ſuch as have no other common Meaſure but an Unit.

8. Numbers compoſed to one another, are ſuch as have ſome Number for their common Meaſure.

So 2, 3, 5, 7, are prime; 4, 6, 9, 10, 12, 14, 15, and all Numbers leſs than 10000, that are not in the following Table, are compoſed Numbers; and 15 and 21 are compoſed to one another, for 3 will meaſure 15 by 5, and 21 by 7; but 15 and 32 (tho' both compoſed Numbers) are prime to one another, for tho' 5 or 3 will meaſure 15, yet neither 5 nor 3 will meaſure 32.

9. A Perfect Number is that which is equal to all its own aliquot Parts taken together.

10. A Diminutive Number is greater than the Sum of all its aliquot Parts.

11. An Abounding Number is leſs than the Sum of its aliquot Parts.

So 6 is a Perfect Number, for the Sum of all its aliquot Parts 1, 2, 3, is 6; but 15 a Diminutive Number, for the Sum of all its aliquot Parts 1, 3, 5, is but 9; and 12 is an Abounding Number, for the Sum of all its aliquot Parts 1, 2, 3, 4, 6, is 16.

12. An Abstract Number is such as hath no Denomination annexed to it.

13. A Contract Number is such as hath a Denomination annexed to it.

So any Number, as 5, is called abstract; but Pounds annexed to the Number 5 (*viz.* 5 Pounds,) makes it contract, and then can signifie nothing but 5 Pounds.

14. A Whole Number is that which numbereth Wholes.

15. A Broken Number or Fraction, numbereth the Parts of an Unit.

16. A Mix'd Number is made by annexing a Fraction to a whole Number.

A Whole Number may be Abstract or Contract, but a Fraction may be wholly Contract, or partly Abstract, and partly Contract; for 2 thirds considered as a Fraction, is abstract, since it is not expressed what it is 2 third parts of; but in respect of its Denomination thirds it is Contract, for it is 2 third Parts, and not 2 fifth Parts, or any other Parts. Lastly, 2 third Parts of a Pound, is Contract, both in respect of its Denomination thirds, and in respect of what they are thirds of.

17. A Decimal Fraction is when the Unit is supposed divided into such a Number of Parts, as may be expressed by an Unit with Cyphers annexed, as 10, 100, 1000, 10000, &c.

18. A Vulgar Fraction admits of any Divsion of the Unit, as into 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or any other Number of Parts.

19. A Vulgar Fraction is written by two Numbers one above the other, with a Line drawn between them, as $\frac{3}{5}$; the uppermost of which is termed the Numerator, because it Numbers the Parts; the lowest the Denomina-

tor, becauſe it denotes what Parts the Unit is divided into. And therefore,

$\frac{2}{3}$	is called	2 third	parts, or	2 thirds.
$\frac{3}{5}$		3 fifth		3 fifths
$\frac{5}{11}$		5 eleventh		5 elevenths.
$\frac{2}{9}$		2 ninth		2 ninths.

But ſince 3 fifths are equal to one fifth, and one fifth, and one fifth, or a fifth of one, and a fifth of one, and a fifth of one, that is, a fifth of three ones; it follows, that 3 fifths are equal to a fifth of 3, or 3 divided by 5; and ſo the foregoing Fractions may be read, A third of 2, a fifth of 3, an eleventh of 5, a ninth of 2; or 2 divided by 3, 3 divided by 5, 5 divided by 11, 2 divided by 9.

20. A Single Fraction is ſuch, as denotes a Part or Parts of ſome Whole.

21. A Compound Fraction is ſuch, as denotes a Part or Parts of ſome Part or Parts.

So $\frac{2}{5}$ and $\frac{3}{7}$ are ſingle Fractions; but $\frac{2}{5}$ of $\frac{3}{7}$, and $\frac{2}{5}$ of $\frac{1}{3}$ of $\frac{3}{8}$ are compound Fractions.

22. A Proper Fraction is leſs than an Unit.

23. An Improper Fraction is greater than an Unit.

SECT. II.

THEOREMS.

1. If the two right-hand Figures of any Number are meaſurable by 4, the whole Number is meaſurable by 4.

2. If the three right-hand Figures of any Number are meaſurable by 8, the whole is meaſurable by 8.

3. If the Sum of the Figures constituting any Number, is a Multiple of 3 or 9, then the whole Number is accordingly a Multiple of 3 or 9.

4. If the right-hand place of any Number be a Cypher, or 5, the whole is a Multiple of 5.

5. If the right-hand place of any Number be a Cypher, the whole is a Multiple of 10.

6. If a Number cannot be divided by some prime Number, not greater than the Square Root thereof, that Number is a Prime.

7. If two Fractions are equal to one another, the Numerator of the one is in such Proportion to its Denominator, as the Numerator of the other to its Denominator.

8. If the Numerator of a Fraction be less than the Denominator, that Fraction is a proper one.

9. But if the Numerator be equal or grearer than the Denominator, the Fraction is improper.

10. Every whole Number may be expressed in the Form of a vulgar Fraction, without altering its Value, if that whole Number be the Numerator, and Unity the Denominator.

SECT. III.

To find all the PRIMES *and the* COMPONENTS *of all the* COMPOSITS *under any given Number. Suppose* 1000.

FROM such a Table as the following, the Problem may be easily solved; for if any Number under that given be sought in the Table, thus, *viz.* the Hundreds at the Top or Bottom, and the Tens and Units in the Margin, at the

Angle of Meeting (which let be called the place of that Number) is found *p*, if the Number be a Prime; or its least Component, if the Number be Composit, which if it be dashed, is the Square Root of the Number.

Then divide the Number by its least Component, and use the Quote in like manner.

So 829 is found to be a Prime.

And so 483 is found to be divisible by 3; and 483 divided by 3, quotes 161, which is by the Table a Multiple of 7; and consequently 161 divided by 7, quotes 23, which by the Table is Prime; whence the Components of 483, are 3, 7, 23; but the Place of 841 gives 29 dashed for a Measure, which shews the Quote will be 29; and therefore 841 is composed of 29 and 29.

And thus you may examine all the Numbers under 1000, the Table reaching only so far.

All even Numbers must be first divided by 2 continually, as oft as possible, and all Numbers ending in 5, divided by 5 as oft as possible, and the Results then sought in the Table, as before.

So 616 divided by 2 three times, quotes 77, which by the Table gives 7 for a Measure, and so the Quote is 11, which by the Table is Prime; consequently the Components of 616, are 2, 2, 2, 7, 11.

And so 435 divided by 5, quotes 87, which by the Table gives 3 for a Measure, and so the Quote 29 is a Prime; whence 435 is composed of 5, 3, 29.

But the Calculating of such a Table being not difficult, I shall next shew the Method to make it.

Let the Paper be first ruled with as many Columns as the limiting Number contains Hundreds, then number the Hundreds at Top and Bot-

	0	1	2	3	4	5	6	7	8	9	
1		p	3	7	p	3	p	p	3	17	1
3	p	p	7	3	13	p	3	19	11	3	3
7	p	p	3	p	11	3	p	7	3	p	7
9	3	p	11	3	p	p	3	p	p	3	9
11	p	3	p	p	3	7	13	3	p	p	11
13	p	p	3	p	7	3	p	23	3	11	13
17	p	3	7	p	3	11	p	3	19	7	17
19	p	7	3	11	p	3	p	p	3	p	19
21	3	11	13	3	p	p	3	7	p	3	21
23	p	3	p	17	3	p	7	3	p	13	23
27	3	p	p	3	7	17	3	p	p	3	27
29	p	3	p	7	3	23	17	3	p	p	29
31	p	p	3	p	p	3	p	17	3	7	31
33	3	7	p	3	p	13	3	p	7	3	33
37	p	p	3	p	19	3	7	11	3	p	37
39	3	p	p	3	p	7	3	p	p	3	39
41	p	3	p	11	3	p	p	3	29	p	41
43	p	11	3	7	p	3	p	p	3	23	43
47	p	3	13	p	3	p	p	3	7	p	47
49	7	p	3	p	p	3	11	7	3	13	49
51	3	p	p	3	11	19	3	p	23	3	51
53	p	3	11	p	3	7	p	3	p	p	53
57	3	p	p	3	p	p	3	p	p	3	57
59	p	3	7	p	3	13	p	3	p	7	59
61	p	7	3	19	p	3	p	p	3	31	61
63	3	p	p	3	p	p	3	7	p	3	63
67	p	p	3	p	p	3	23	13	3	p	67
69	3	13	p	3	7	p	3	p	11	3	69
71	p	3	p	7	3	p	11	3	13	p	71
73	p	p	3	p	11	3	p	p	3	7	73
77	7	3	p	13	3	p	p	3	p	p	77
79	p	p	3	p	p	3	7	19	3	11	79
81	3	p	p	3	13	7	3	11	p	3	81
83	p	3	p	p	3	11	p	3	p	p	83
87	3	11	7	3	p	p	3	p	p	3	87
89	p	3	17	p	3	19	13	3	7	23	89
91	7	p	3	17	p	3	p	7	3	p	91
93	3	p	p	3	17	p	3	13	19	3	93
97	p	p	3	p	7	3	17	p	3	p	97
99	3	p	13	3	p	p	3	17	29	3	99
	0	1	2	3	4	5	6	7	8	9	

Bottom

Bottom, and the Tens and Units in the Margins, as in the Example before you.

First, Then in the place of the first odd Prime 3, put *p*; and then raise 3 to all its Powers under 1000, the Number given; and in the place of 9, its Square, place 3 with a Dash, thus 3'; but in the Places of all its superior Powers, put 3 without the Dash.

Secondly, The next odd Number is 5, which is passed by, because all its Multiples end in 5.

Thirdly, The next odd Number is 7, whose place is yet vacant, and is therefore to have *p* in it; then multiply the Numbers belonging to every place that is filled, by 7 (except such as give Products greater than the limiting Number,) and in the place for the Product, put the Figure that was in the place of the Multiplicand, without a Dash, if it had one; or else the Multiplicand itself, if it be a Prime. Then in the place of the Square of 7, put 7 with a Dash, as 7'; and by that Square multiply the same Multiplicands, putting in the places of the Products as before. But in the places of the Powers of 7, put 7 without a Dash; and by these Powers multiply the same Multiplicand, putting in the places of the Products as before.

Fourthly, The next odd Number is 9, whose place is filled, and therefore to be pass'd over.

Fifthly, The next odd Number is 11, whose place is yet vacant, and is therefore to have *p* in it; then by 11, multiply the Numbers belonging to every place that is filled, except such as give Products greater than the limiting Number, and in the place for the Product, put the Figure that was in the place of the Multiplicand, without a Dash, if it had one, or else the Multiplicand itself, if it be a Prime; then in the place of the Square of 11, put 11 with a Dash, and by the

the Square and the other Powers of 11, multiply the ſame Multiplicands, putting in the places of the Products as before; and ſo uſe 13, 17, 19, 23, &c. omitting 15, 21, 25, 27, &c.

This Table is continued to 100000 in Dr. *Pell*'s Edition of *Brancker*'s Algebra, and from thence tranſcribed into Dr. *Harris*'s Lexicon, *Vol.* 2.

SECT. IV.

To reduce any Number given, into its COMPONENT PRIMES.

The RULE.

DIvide the Number given by 2 continually as oft as poſſible, without a Remainder, and ſo the Reſult by 3 as oft as poſſible, and this laſt Reſult by 5 as oft, and ſo on by 7, 11, 13, &c. all the Prime Numbers, till there ariſe a Quotient leſs than the Diviſor; and then the Diviſors and the laſt Dividend are the Primes compoſing the Number given. *e.g.*

Reduce 122760, to its component Primes.

```
 2) 122760
  2) 61380
  2) 30690
  3) 15345
   3) 5115
   5) 1705
   11) 341
        31
```

Anſwer 2, 2, 2, 3, 3, 5, 11, 31.

In the Operation I have omitted the Diviſion by 7, becauſe it would not meaſure 341; and

 after-

afterwards omitted dividing by 13 and 17, because they are no aliquot Parts of 31.

Reduce 360 to its constituent Primes.

2) 360
2) 180
2) 90
3) 45
3) 15
5

Answer 2, 2, 2, 3, 3, 5.

SECT. V.

To find all the just DIVISORS *to any Number.*

The RULE.

MAKE as many Columns as there are different Component Primes in the given Number: And at the Head of each Column, place each Prime, and where there are more than one Prime of the same sort, place in the same Column the Powers of that Prime, till the Index of the last Power be the Number of those like Component Primes; lastly, multiply every Number in the first Column, by every Number in the second, placing each Product in the second; and then every Number in the first and second by every Number in the third, placing each Product in the third, and so with the 4*th*, 5*th*, &c. Columns, till all are thus used.

And so shall all the Numbers in these several Columns, be all the Divisors; *e.g.*

To

To find all the just Divisors of 360.

2	360	2	3	5
2	180	4	9	10
2	90	8	6	20
3	45		12	40
3	15		24	15
	5		18	45
			36	30
			72	60
				120
				90
				180
				360

Answer, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360.

SECT. VI.

To find all the ALIQUOT PARTS *of any Number given.*

FIRST, by *Sect.* 5. find all the just Divisors of the Number given, excluding the Number itself, which Divisors with Unity, are all the aliquot Parts.

So all the aliquot Parts of 360, are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180.

SECT. VII.

To find as many PERFECT NUMBERS *as you please.*

TAKE any ſuch Power of 2, as being leſſened by an Unit, leaves a Prime Number, and multiply that Remainder by half that Power, and the Product is a perfect Number.

	Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.
2	4	8	16	32
	1	1	1	1
	3	7	15	31
	2	4		16
	6	28		496

Ex. 5.	Ex. 6.	Ex. 7.
64	128	256
1	1	1
63	127	255
	64	
	508	
	762	
	8128	

So 6, 28, 496, 8128, are perfect Numbers: But becauſe 15, 63, 255, are compoſed Numbers, the Reſults that would ariſe by multiplying them by 8, 32, 128, would be abounding Numbers.

SECT.

SECT. VIII.

Two Numbers being given, to find their greatest COMMON MEASURE.

The RULE.

DIvide the greater by the leſſer, and if nothing remain, that leſſer Number is the greateſt common Meaſure required; but if there be a Remainder, divide the laſt Diviſor by that Remainder; and if there be ſtill a Remainder, divide the laſt Diviſor by the laſt Remainder; and ſo proceed till you come to a Diviſion without a Remainder; and then the laſt Diviſor is the greateſt common Meaſure.

Ex. 1. What is the greateſt common Meaſure of 612, and 540?

```
540) 612 (1
      72) 540 (7
           36) 72 (2        Anſwer 36.
               ..
```

But the foregoing Operation may be conveniently placed, thus.

612	1
540	7
72	2
36	
..	

Ex. 2.

Ex. 2. What is the greatest common Measure of 720 and 336?

336) 720 (2
48) 336 (7   Or thus,
...

720	2
336	7
48	

Answer, 48.

Ex. 3. What is the greatest common Measure of 360 and 24?

24) 360 (15
120   Or thus,
...

360	15
24	
120	
...	

Answer 24.

Ex. 4. What is the greatest common Measure of 643536 and 4500?

4500) 643536 (143
19353
13536
36) 4500 (125
90
180
...

Or thus,

643536	143
4500	125
19353	
13536	
36	
90	
180	
...	

Ex. 5.

Ex. 5. What is the greatest common Measure of 3187 and 1111?

```
1111)3187(2
     965)1111(1
          146)965(6
               89)146(1
                   57)89(1
                      32)57(1
                         25)32(1
                             7)25(3
                               4)7(1
                                 3)4(1
                                   1)3(3
```

Or thus,

3187	3
1111	1
965	6
146	1
89	1
57	1
32	1
25	3
7	1
4	1
3	3
1	

The Numbers are prime to one another, for the Answer is 1.

SECT.

SECT. IX.

Two or more Numbers being given, to find a COMMON MULTIPLE *to them.*

The RULE.

MULTIPLY all the Numbers continually, and the Product is the Number sought.

Ex. Find a common Multiple to 3, 5, 6, and 9.

$$\begin{array}{r} 3 \\ 5 \\ \hline 15 \\ 6 \\ \hline 90 \\ 9 \\ \hline 810 \end{array}$$

Answer 810.

SECT. X.

Two Numbers being given, to find their least COMMON MULTIPLE.

The RULE.

FIRST find their greatest common Measure, then divide either of the Numbers by that Measure, and multiply the Quotient by the other of those Numbers, and the Product is the Number sought.

Ex.

Ex. What is the leaſt common Multiple of 24 and 32?

32	1	8)32(	4
24	3	..	24
8			96 The Anſwer.

Ex. 2. What is the leaſt common Multiple of 18 and 49?

49	2		
18	1	1)49(	49
13	2		18
5	1		392
3	1		49
2			882 The Anſwer.
1			

Hence it appears, that the leaſt common Multiple to two Numbers prime to one another is their Product.

SECT. II.

Three or more Numbers being given, to find their leaſt COMMON MULTIPLE.

The RULE.

FIrſt find the leaſt common Multiple to two of them, and then to this Multiple, and the third Number, find the leaſt common Multiple; then to the laſt Multiple and the *4th* Number propoſed, find the leaſt common Multiple; and ſo proceed with the *5th*, *6th*, &c. given Numbers,

till the laſt is thus uſed; And ſo the Multiple thus found ſhall be that required.

Ex. 1. What is the leaſt Multiple to 8, 12, 9, 15?

```
12|1        24|2        72|4
 8|1         9|1        15|1
 4|          6|2        12|4
             3|          3|

4) 12 (3
       8    3) 24 (8    3) 72 (24
      --           9           15
      24          --          ---
                  72          120
                               24
                              ---
                              360
```

The Anſwer.

But if either of the Numbers propoſed is an aliquot Part of another of them, that aliquot Part may be omitted in the Work.

Ex. 2. What is the leaſt common Multiple to 4, 3, 6, 8, 12, 9?

```
12|1        24|2
 8|2         9|1
 4|          6|2
             3|

4) 8 (2     3) 24 ( 8
      12             9
      --            --
      24            72  The Anſw.
```

Here

Here 4, 3, and 6 are omitted in the Operation, because 4 and 3 and 6, are aliquot Parts of 12.

If the least common Multiple to some of the preceding Numbers, is also a Multiple to some of the following Numbers; such following Numbers may be omitted in the Operation.

Ex. 3. Find the least common Multiple to 15, and 8, and 12.

15	1	120	10
8	1	12	
7	7		
1			

$$\begin{array}{r} 15 \\ 8 \\ \hline 120 \end{array}$$

Answer 120.

The Work of this Section may be otherwise performed thus.

If the second be composed to the first, divide the 2*d* by their greatest common Measure, and cancel it, and place the Quote beneath it; if the 3*d* be composed to either or both of the preceding uncancell'd Figures, divide it by their greatest common Measure continually, placing the Quotes beneath it, and cancel it and all its Quotes except the last; and so if the 4*th* be composed to any or all the preceding uncancell'd Figures, divide it by their greatest common Measures continually, placing the Quotes beneath it under one another, and cancelling it and all its Quotes but the last; and thus proceed till you have thus used all the Numbers proposed. And then the Product made by the continual Multiplication of all those uncancelled Numbers, is the Number sought.

But

But in this Method likewise, the Numbers that are aliquot Parts of others may be omitted.

And so the Work of the two first Examples will stand thus.

$$\begin{array}{lllll}
\textit{Ex. } 1. & 8 & \cancel{12} & \cancel{9} & \cancel{15} \\
 & & 3 & 3 & 5
\end{array}
\qquad
\begin{array}{r}
8 \\
\underline{3} \\
24 \\
\underline{3} \\
72 \\
\underline{5} \\
360
\end{array}$$

The Answer.

$$\begin{array}{lllllll}
\textit{Ex. } 2. & \cancel{4} & \cancel{3} & \cancel{6} & 8 & \cancel{12} & \cancel{9} \\
 & & & & & 3 & 3
\end{array}
\qquad
\begin{array}{r}
8 \\
\underline{3} \\
24 \\
\underline{3} \\
72
\end{array}$$

The Answer.

CHAP.

CHAP. II.

Reduction of *Fractions.*

SECT. I.

To Reduce FRACTIONS *to their least Terms.*

The RULE.

DIVIDE both Terms by their greatest common Measure, and the Fraction formed by their Quotients, is the Answer.

Reduce $\frac{540}{612}$ to its least Terms.

612	1
540	7
72	2
36	
..	

$$36)\ \frac{540}{612}\left(\frac{15}{17}\right.\ \text{The Answer.}$$

180

252

Reduce

Reduce $\frac{1111}{3187}$ to its least Terms.

3187	3
1111	1
965	6
146	1
89	1
57	1
32	1
25	3
7	1
4	1
3	3
1	

$1)\frac{1111}{3187}(\frac{1111}{3187}$ The Answer.

Hence if the Terms are prime to one another, the Fraction is in its least Terms already.

SECT. II.

To reduce compound to single FRACTIONS.

The RULE.

MULTIPLY all the Numerators together continually, and the Product is the Numerator of the Fraction sought; and all the Denominators together for the Denominator.

Reduce $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of $\frac{2}{3}$ to a single Fraction.

1	2
5	6
5	12
3	4
15	48
2	3
30	144

Answer $\frac{30}{144}$.

If

If the Anſwer be required in its leaſt Terms: Obſerve, that if any Numerator be equal to any Denominator, reject them, and conceive Unity in their ſtead: Or if any Numerator be compoſed to any Denominator, divide ſuch Numerator and Denominator by their greateſt common Meaſure, and uſe the Quotients in their ſtead.

And ſo the Work of the foregoing Example will ſtand thus.

$$\frac{1}{\cancel{2}} \text{ of } \frac{5}{6} \text{ of } \frac{\cancel{3}}{4} \text{ of } \frac{\cancel{2}}{\cancel{3}} \qquad \begin{array}{cc} 1 & 6 \\ \underline{5} & \underline{4} \\ 5 & 24 \end{array}$$

Anſwer $\frac{5}{24}$.

Reduce $\frac{15}{16}$ of $\frac{2}{7}$ of $\frac{14}{25}$ to the leaſt ſingle Fraction.

$$\begin{array}{ccccc} & & 1 & & \\ & & \cancel{2} & & \\ 3 & 1 & & 3 & 4 \\ \frac{\cancel{15}}{\cancel{16}} \text{ of} & \frac{\cancel{2}}{\cancel{7}} \text{ of} & \frac{\cancel{14}}{\cancel{25}} & \frac{1}{3} & \frac{5}{20} \\ 8 & 1 & 5 & & \\ 4 & & & & \end{array}$$

Anſwer $\frac{3}{20}$.

In this Example 15 is compoſed to 25, and 5 is their greateſt common Meaſure, and ſo the Quotients are 3 and 5; 2 is compoſed to 16, and 2 is their greateſt common Meaſure, and ſo 1 and 8 are the Quotients; 14 and 7 are compoſed to one another, and 7 is their greateſt common Meaſure, and ſo the Quotients are 2 and 1; laſtly, the 2 over the 14, and the 8 under the 16 are compoſed to one another, and 2 is their great-

eſt

eſt common Meaſure, and ſo the Quotients are 1 and 4; So that now among the Numerators are found uncancell'd 3, 1, 1, and among the Denominators 4, 1, 5, which by the Rule form the Fraction $\frac{3}{20}$, which is the leaſt ſingle Fraction that the foregoing Compound one is reducible to.

Reduce $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{1}{14}$ of $\frac{7}{25}$ of 24, to the leaſt ſingle Fraction poſſible.

$$\frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\cancel{5}}{\cancel{6}} \text{ of } \frac{\cancel{3}}{\underset{2}{\cancel{8}}} \text{ of } \frac{1}{\underset{2}{\cancel{14}}} \text{ of } \frac{\cancel{7}}{\underset{5}{\cancel{25}}} \text{ of } \frac{\overset{4}{\cancel{24}}}{1}$$

Anſwer $\frac{1}{12}$.

If it be objected that the finding of the greateſt common Meaſure ſo often is more Trouble than to form the Fraction after the uſual Method, and then at one Operation to find the greateſt common Meaſure, and ſo reduce it to its leaſt Terms; it may be anſwered, that ſuch greateſt common Meaſures in ſmall Numbers (as moſt Fractions are uſually propoſed in before they are moulded together) appear upon View to any one that is but a little acquainted with Numbers; but ſome common Meaſures, if not the greateſt, will eaſily appear even in large Numbers moſt commonly; and by uſing thoſe Meaſures, the Quotes are ſmaller, and ſo other Meaſures may appear, and thus very often the leaſt Quotes may be found; or at leaſt the Fraction will hereby be found in lower Terms.

Indeed

Indeed if no ſuch common Meaſure do appear upon View, the beſt way is to work as uſual, and then to reduce it as in the laſt Section.

There is a ſort of Fractions often come to hand, whoſe Terms are Fractions or mix'd Numbers, as $\dfrac{72\frac{12}{145}}{19\frac{7}{12}}$, and might properly be called Compound ones, if that Name had not been already given to the foregoing ſort: How to uſe ſuch, you will find at the *1ſt* Section of the *7th* Chapter.

SECT. III.

To reduce mix'd Numbers to improper FRACTIONS.

The RULE.

MUltiply the whole Number by the Denominator of the Fraction; and to the Product add the Numerator, and the Sum is the Numerator of the Fraction ſought, and the given Denominator is the Denominator ſought.

Reduce $47\frac{3147}{8400}$ to an improper Fraction.

$$\begin{array}{r} 47 \\ 8400 \\ \hline 18800 \\ 376 \\ 3147 \\ \hline 397947 \end{array} \quad \text{Anſwer } \frac{397947}{8400}.$$

SECT. VI.

To reduce improper FRACTIONS *to whole or mix'd Numbers.*

The RULE.

DIvide the Numerator by the Denominator, and the Quotient is the whole Number; and if there be a Remainder, it is the Numerator of the Fraction to be annexed, whose Denominator is the Denominator given.

Reduce $\frac{397947}{8400}$ to its equivalent whole or mix'd Number.

8400) 397947 (47
619
3147 Answer $47\frac{3147}{8400}$.

Reduce $\frac{781}{11}$ to its equivalent whole or mix'd Number.

11) 781 (71
11
•• Answer 71.

If the fractional Part of the Answer be not in its least Terms, it may be convenient to reduce it to such; for the ready Performance whereof, observe the following Work.

Reduce

Reduce $\frac{1690}{221}$ to its equivalent whole or mix'd Number in its least Terms.

$$\begin{array}{r|l} 1690 & 7 \\ 221 & 1 \\ 143 & 1 \\ 78 & 1 \\ 65 & 5 \\ 13 & \\ \cdot\cdot & \end{array} \qquad \begin{array}{c} \cdot\cdot \\ 13 \\ 13)\frac{143}{221}(\frac{11}{17} \\ 91 \\ \cdot\cdot \end{array}$$

Answer $7\frac{11}{17}$.

SECT. V.

To reduce a FRACTION *to any Denomination its capable of, without a mix'd Number, or a* FRACTION *for its Numerator.*

The RULE.

MUltiply the Numerator by the Denominator of the Fraction sought, and divide that Product by the Denominator of the Fraction given; and so the Quotient, if nothing remain, shall be the Numerator sought; but if there be a Remainder, the Fraction is incapable of that Denomination without a mix'd Number, or a Fraction for its Numerator.

Reduce $\frac{18}{135}$ to another equal to it, whose Denominator may be 225.

$$\begin{array}{r} 18 \\ \underline{225} \\ 1800 \\ \underline{225} \\ 135)4050(30 \\ \cdots0 \end{array}$$

Answer $\frac{30}{225}$.

E 2 Reduce

Reduce $\frac{6}{8}$ to another equal to it whoſe Denominator may be 27.

```
      6
     27
 8)162(20
     .2
```

From whence it appears that the Numerator will be a mix'd Number, for the full Quote is $20\frac{2}{8}$, and ſo the Fraction ſought muſt be $\frac{20\frac{2}{8}}{27}$.

If the Fraction be in its leaſt Terms, as it ought to be, then divide the Denominator of the Fraction ſought, by the Denominator of the Fraction given, and (if there be no Remainder, the Fraction is capable of the required Denomination, otherwiſe not) then multiply that Quotient by the Numerator of the Fraction given; and ſo the Product ſhall be the Numerator of the Fraction ſought.

Reduce $\frac{2}{15}$ to another Fraction equal to it whoſe Denominator ſhall be 225.

```
15)225(15
    75  2
    ..  --
        30     Anſwer 30/225.
```

This Method may be ſometimes uſed, when the Fraction is not in its loweſt Terms, *viz.* When the Denominator of the given Fraction is an aliquot Part of the Denominator of that ſought.

Reduce $\frac{42}{76}$ to another equal to it, whoſe Denominator ſhall be 608.

76

```
76) 608 ( 8
 ...  42
 336        Answer 336/608.
```

But if the Denominator of the given Fraction be an aliquant Part of the other Denominator, and the given Fraction not in its least Terms; The last Method will not so easily give us the Numerator of the Fraction sought, tho' it might be had by the first of this Section; as may appear by the first Example.

SECT. VI.

To Reduce FRACTIONS, *having unequal Denominators, to others equal in Value, having equal Denominators; usually called reducing them to common Denominators.*

The RULE.

MUltiply all the Denominators together continually, and the Product is the common Denominator, and each Numerator by all the Denominators except its own, and the Products are the required Numerators.

Reduce $\frac{7}{12}$ $\frac{5}{8}$ $\frac{4}{9}$, to others equal to them, having equal Denominators.

```
 12     7     5     4
  8     8    12    12
 --    --    --    --
 96    56    60    48
  9     9     9     8
---   ---   ---   ---
864   504   540   384
```

Answer $\frac{504}{864}$, $\frac{540}{864}$, $\frac{384}{864}$.

If

If the Fractions to be reduced are many, after the common Denominator is found, the Numerators are best found by the Doctrine of the *6th* Section.

Reduce $\frac{6}{7}$ $\frac{3}{5}$ $\frac{4}{9}$ $\frac{7}{11}$ $\frac{5}{13}$ $\frac{9}{14}$ to others equal to them having equal Denominators.

7	7) 630630	9) 630630
5	90090	70070
35	6	4
9	540540	280280
315		
11	5) 630630	11) 630630
3465	126126	57330
13	3	7
10395	378378	401310
3465		
45045	13)630630 (48510	
14	110 5	
180180	66 242550	
45045	13	
630630	14)630630 (45045	
	70 9	
	63 405405	

Answer $\frac{540540}{630630}$, $\frac{378378}{630630}$, $\frac{280280}{630630}$, $\frac{401310}{630630}$, $\frac{242550}{630630}$, $\frac{405405}{630630}$.

SECT. VII.

To reduce FRACTIONS, *in their least Terms, to their least common Denominators.*

The RULE.

FIRST, by the 11*th* Section of the first Chapter, find the least common Multiple to the Denominators, which shall be the least common

mon Denominator; and then, by the 5*th* Section of this Chapter, find the Numerators.

Reduce $\frac{5}{8}$ $\frac{7}{12}$ $\frac{4}{9}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{1}{4}$ to the least common Denominator possible.

$$\begin{array}{r|l} 12 & 1 \\ 8 & 2 \\ 4 & \end{array} \qquad \begin{array}{r} 4)\,12\,(3 \\ 8 \\ \hline 24 \end{array} \qquad \begin{array}{r|l} 24 & 2 \\ 9 & 1 \\ 6 & 2 \\ 3 & \end{array} \qquad \begin{array}{r} 3)\,24\,(8 \\ 9 \\ \hline 72 \end{array}$$

So 72 is the least common Multiple to the Denominator.

Or thus,

$$\begin{array}{cccccccc} 8 & \cancel{12} & \cancel{9} & \cancel{3} & \cancel{6} & \cancel{4} & & 8 \\ & 3 & 3 & & & & & 3 \\ \cline{8-8} & & & & & & & 24 \\ & & & & & & & 3 \\ \cline{8-8} & & & & & & & 72 \end{array}$$

as before.

Whence the least common Denominator is 72.

$$\begin{array}{r} 8)72(9 \\ 5 \\ \hline 45 \end{array} \qquad \begin{array}{r} 12)72(6 \\ 7 \\ \hline 42 \end{array} \qquad \begin{array}{r} 9)72(8 \\ 4 \\ \hline 32 \end{array} \qquad \begin{array}{r} 3)72(24 \\ 2 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 6)72(12 \\ 5 \\ \hline 60 \end{array} \qquad \begin{array}{r} 4)72(18 \\ 1 \\ \hline 18 \end{array}$$

Answer $\frac{45}{72}$ $\frac{42}{72}$ $\frac{32}{72}$ $\frac{48}{72}$ $\frac{60}{72}$ $\frac{18}{72}$.

In all the Operations of this Chapter, I have set down every single Multiplication or Division, to explain the Methods herein made use of; tho' indeed many of them, without any Burthen to the Memory, may be omitted in Practice; an Instance whereof shall be taken from this last Example.

First,

First, Any one that is but a little skilled in the preceding Sections, must know, that 4 is the greatest common Measure of 8 and 12, and that 12 divided by 4 quotes 3, which multiplied by 8 produces 24, which is the least common Multiple of 8 and 12.

And also that 3 is the greatest common Measure of 9 and 24; wherefore since 24 divided by 3 quotes 8, and 8 multiplied by 9 produces 72; it follows that 72 is the least common Multiple of 8, 12, 9, and therefore of 3, 6, 4 also; because these are aliquot Parts of 12; consequently, by the Rule, 72 is the least common Denominator.

And then for the Numerators; *first*, 72 divided by 8 quotes 9, which multiplied by 5 produces 45, the Numerator of the first: And in like manner may the Numerator of every Fraction, and consequently every Fraction be formed, without so much as the writing of one single Digit.

And generally all Fractions, that are expressed in small Terms, may be thus reduced.

Hence it appears that the keeping of Fractions in their least Terms, is a very great Advantage in future Operations.

CHAP.

CHAP. III.

Addition of Vulgar FRACTIONS.

SECT. I.

To collect, or add two or more FRACTIONS *together.*

The RULE.

REDUCE the given Fractions, if compound, to single ones, and then to a common Denominator, if they have not such already; then add up their Numerators, and to the Sum subscribe the common Denominator, which gives the Answer. Then reduce it to its least Terms, and if improper, to its equivalent whole or mix'd Number.

Add $\frac{2}{13}$, $\frac{3}{13}$ and $\frac{7}{13}$ together.

$$\left.\begin{array}{r}2\\3\\\underline{7}\end{array}\right\}\text{ the Numerators.}$$

12 Answer $\frac{12}{13}$.

Add $\frac{3}{16}$ and $\frac{5}{16}$ together.

$$\begin{array}{r} 3 \\ 5 \\ \hline 8 \end{array}$$ Anſwer $\frac{8}{16}$, or in its leaſt Terms $\frac{1}{2}$.

Add $\frac{2}{7}$ and $\frac{3}{7}$ and $\frac{5}{7}$ together.

$$\begin{array}{r} 2 \\ 3 \\ 5 \\ \hline 10 \end{array}$$ Anſwer $\frac{10}{7}$, or its equal $1\frac{3}{7}$.

Add $\frac{1}{4}$ and $\frac{2}{9}$ and $\frac{4}{7}$ together; which reduced, are $\frac{63}{252}$, $\frac{56}{252}$, $\frac{144}{252}$.

$$\begin{array}{r} 63 \\ 56 \\ 144 \\ \hline 263 \end{array}$$ Anſwer $\frac{263}{252}$, or in a mix'd Number $1\frac{11}{252}$.

Add $\frac{3}{4}$ of $\frac{5}{6}$ to $\frac{2}{3}$ and $\frac{4}{5}$ and $\frac{2}{9}$.

After the improper Fraction is reduced, they are $\frac{5}{8}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{2}{9}$, and theſe reduced to a common Denominator, are $\frac{225}{360}$, $\frac{240}{360}$, $\frac{288}{360}$, $\frac{80}{360}$.

$$\begin{array}{r} 225 \\ 240 \\ 288 \\ 80 \\ \hline 833 \end{array}$$ Anſwer $\frac{833}{360}$, or in a mix'd Number $2\frac{113}{360}$.

SECT.

SECT. II.

To add mix'd NUMBERS *together.*

The RULE.

FIRST add up the fractional Part, as taught in the last Section, and if their Total be a proper Fraction subscribe it, and proceed to add up the Integers; but if the total Sum of the Fractions be an improper one, reduce it to its equivalent mix'd Number, and subscribe the fractional Part thereof, but carry the Integers to be added up with the Integral Parts of the given Numbers.

Add $476\frac{1}{3}$ and $24\frac{3}{4}$ and $74\frac{5}{6}$ and $81\frac{3}{8}$ and $152\frac{5}{14}$ together.

But when the fractional Parts are reduced to a common Denominator, as taught in the last Section of the second Chapter, they will be as followeth.

$$
\begin{array}{rl}
476 & \frac{56}{168} \\[6pt]
24 & \frac{126}{168} \\[6pt]
74 & \frac{140}{168} \\[6pt]
81 & \frac{63}{168} \\[6pt]
152 & \frac{60}{168} \\
\hline
 & \frac{445}{168} \quad \text{Sum of the fractional Parts, which reduced, is } 2\frac{109}{168}. \\
\hline
809 & \frac{109}{168} \quad \text{The Answer.}
\end{array}
$$

SECT. III.

To add to any FRACTION *any Part or Parts of the same* FRACTION.

The RULE.

TO the Numerator of the Fraction, expressing the Part or Parts to be added, add the Denominator, and multiply the Sum by the other Numerator, and this shall be the Numerator of the Answer; lastly, multiply the two Denominators together, and the Product is the Denominator of the Answer.

To $\frac{3}{4}$ add $\frac{2}{7}$ of the same $\frac{3}{4}$.

$$\begin{array}{rcr} 2 & \quad & 7 \\ \underline{7} & & \underline{4} \\ 9 & & 28 \\ \underline{3} & & \\ 27 & & \end{array} \qquad \text{Answer } \tfrac{27}{28}.$$

Or thus; After the Numerator 2 is encreased by its Denominator 7, and so becomes 9, reduce the compound Fraction $\frac{9}{7}$ of $\frac{3}{4}$ to a single one, and it is the Answer; for $\frac{9}{7}$ of $\frac{3}{4}$ gives $\frac{27}{28}$.

To $\frac{3}{4}$ add $\frac{7}{9}$ of the same $\frac{3}{4}$.

$$\begin{array}{r} 7 \\ \underline{9} \\ 16 \end{array} \qquad \tfrac{16}{9} \text{ of } \tfrac{3}{4} \text{ reduced, becomes } \tfrac{4}{3} \text{ or } 1\tfrac{1}{3} \text{ the Answ.}$$

SECT. IV.

To add any Parts of any FRACTION *to any Parts of the same* FRACTION.

The RULE.

FIRST add the Parts, as taught in the first Section of this Chapter; and make of the Sum of the Parts and the Fraction a compound Fraction, which reduced to a single one shall be the Answer.

To $\frac{2}{3}$ of $\frac{5}{7}$ add $\frac{3}{5}$ of $\frac{5}{7}$.

But the Sum of $\frac{2}{3}$ and $\frac{3}{5}$ is $\frac{19}{15}$.

And $\frac{19}{15}$ of $\frac{5}{7}$ reduced, is $\frac{19}{21}$ the Answer.

CHAP.

CHAP. IV.

Subſtraction of Vulgar Fractions.

SECT. I.

To ſubſtract one FRACTION *from another.*

The RULE.

REDUCE the given Fractions, if compound, to ſingle ones, and then to a common Denominator, if they have not ſuch already; laſtly, ſubſtract their Numerators, and to the Remainder ſubſcribe the common Denominator, and it is the Anſwer, which reduce to its leaſt Terms.

From $\frac{5}{8}$ ſubſtract $\frac{3}{8}$.

$$\begin{array}{r} 5 \\ \underline{3} \\ 2 \end{array}$$

Anſwer $\frac{2}{8}$ or in its leaſt Terms $\frac{1}{4}$.

From

From $\frac{5}{6}$ substract $\frac{3}{8}$.

These reduced are $\frac{20}{24}$ and $\frac{9}{24}$.

$$\begin{array}{r} 20 \\ 9 \\ \hline 11 \end{array} \quad \text{Answer } \frac{11}{24}.$$

Substract $\frac{2}{5}$ of $\frac{3}{8}$, or its Equal $\frac{3}{20}$ from $\frac{2}{5}$, but these reduced to a common Denominator are $\frac{8}{20}$ and $\frac{3}{20}$.

And so the Answer is $\frac{5}{20}$, or in its least Terms $\frac{1}{4}$.

From $\frac{2}{3}$ of $\frac{7}{8}$ take $\frac{3}{4}$ of $\frac{2}{9}$; or when reduced to single Fractions, from $\frac{7}{12}$ take $\frac{1}{6}$, which reduced to a common Denominator, are $\frac{7}{12}$ and $\frac{2}{12}$ wherefore the Answer is $\frac{5}{12}$.

SECT. II.

To Substract MIX'D NUMBERS.

The RULE.

FIRST substract the Fractions according to the Doctrine of the foregoing Section, and then the Integers: But if the fractional Part of the Subtrahend be greater than the fractional Part of the Minuend (and consequently when reduced to a common Denominator, the Numerator of the former, greater than the Numerator of the latter;) then substract the Numerator of the Subtrahend from the common Denominator, and to the Remainder add the Numerator of the Minuend, and the Sum is the Numerator to the fractional Part of the Answer, to which subscribe the

the common Denominator; then carry one to the integral Part of the Subtrahend, and substract.

From $17\frac{2}{3}$ substract $12\frac{3}{5}$; or when the Fractions are reduced to a common Denominator, from $17\frac{10}{15}$ take $12\frac{9}{15}$.

$$\begin{array}{l} 17\frac{10}{15} \\ \underline{12\frac{9}{15}} \\ 5\frac{1}{15} \quad \text{The Answer.} \end{array}$$

From $27\frac{2}{5}$ substract $15\frac{3}{4}$, or from $27\frac{8}{20}$ take $15\frac{15}{20}$.

$$\begin{array}{l} 27\frac{8}{20} \\ \underline{15\frac{15}{20}} \\ \quad 5 \\ \underline{\quad 8} \\ 11\frac{13}{20} \quad \text{The Answer.} \end{array}$$

From $14\frac{2}{9}$ substract $\frac{5}{7}$.

These reduced are $\left\{\begin{array}{l} 14\frac{14}{63} \\ \underline{\quad \frac{45}{63}} \\ \quad 18 \\ \underline{\quad 14} \\ 13\frac{32}{63} \quad \text{The Answer.} \end{array}\right.$

From 14 substract $4\frac{5}{8}$.

$$\begin{array}{l} 14 \\ \underline{\ 4\frac{5}{8}} \\ \ 9\frac{3}{8} \quad \text{The Answer.} \end{array}$$

From $14\frac{5}{8}$ substract 5.

$$\begin{array}{l} 14\frac{5}{8} \\ \underline{\ 5} \\ \ 9\frac{5}{8} \quad \text{The Answer.} \end{array}$$

From

From 1 substract $\frac{7}{11}$.

$$\begin{array}{l} 1 \\ -\frac{7}{11} \\ \hline \frac{4}{11} \end{array}$$ The Answer.

From 7 substract $\frac{5}{9}$.

$$\begin{array}{l} 7 \\ -\frac{5}{9} \\ \hline 6\frac{4}{9} \end{array}$$ The Answer.

SECT. III.

From any FRACTION *to substract any Parts of the same.*

The RULE.

FROM the Denominator of that Fraction which denotes the Parts to be taken away, substract its Numerator, and multiply the Remainder by the other Numerator, and this shall be the Numerator of the Answer; lastly, multiply the two Denominators together, and the Product is the Denominator of the Answer.

From $\frac{5}{7}$ substract $\frac{3}{10}$ of $\frac{5}{7}$.

$$\begin{array}{rr} 10 & 10 \\ 3 & 7 \\ \hline 7 & 70 \\ 5 & \\ \hline 35 & \end{array}$$ Answer $\frac{35}{70}$ or in its least Terms $\frac{1}{2}$.

Or thus, after the Denominator 10 is lessened by its Numerator 3, and the Difference 7 made

G Nume-

rator inſtead of the 3; reduce the compound Fraction $\frac{7}{10}$ of $\frac{5}{7}$ to a ſingle one, and it gives $\frac{1}{2}$ the Anſwer as before.

SECT. IV.

From any Parts of a FRACTION *to ſubſtract any other Parts of the ſame.*

The RULE.

FIRST ſubſtract the Parts, and with the Remainder and the propoſed Fraction form a compound one, which reduced to a ſingle one is the Anſwer.

From $\frac{3}{5}$ of $\frac{4}{7}$ ſubſtract $\frac{5}{11}$ of the ſame $\frac{4}{7}$.

If $\frac{5}{11}$ be ſubſtracted from $\frac{3}{5}$ the Remainder is $\frac{8}{55}$, and $\frac{8}{55}$ of $\frac{4}{7}$ reduced is $\frac{32}{385}$ the Anſwer.

CHAP.

CHAP. V.

Multiplication of vulgar Fractions.

The RULE.

REDUCE compound Fractions to single ones, and mix'd Numbers to improper Fractions, and express Integers in the Form of Fractions; then multiply the Numerators of the Factors together, and the Product is the Numerator of the Answer; and likewise the Denominators together, and the Product is Denominator of the Answer.

But observe that the Terms of the Factors may be lessened, by the Directions laid down in *Sect.* 3. of *Chap.* II. and so the Answer will be in its least Terms, which if an improper Fraction, it is sometimes convenient to reduce to its equivalent whole, or mix'd Number.

Example 1. Multiply $\frac{4}{7}$ by $\frac{5}{9}$.

$$\begin{array}{rr} 4 & 7 \\ \underline{5} & \underline{9} \\ 20 & 63 \end{array} \quad \text{Answer } \frac{20}{63}.$$

Ex. 2. Multiply $\frac{9}{14}$ by $\frac{16}{21}$.

$$\frac{\overset{3}{\cancel{9}}}{\underset{7}{\cancel{14}}} \quad \frac{\overset{8}{\cancel{16}}}{\underset{7}{\cancel{21}}} \quad \frac{3}{8}\ \frac{8}{24} \quad \frac{7}{7}\ \frac{7}{49}$$

Answer $\frac{24}{49}$.

Ex. 3. Multiply $\frac{2}{3}$ by $\frac{3}{5}$ of $\frac{5}{8}$.

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \text{ by } \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \text{ of } \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}}$$

Answer. $\frac{1}{4}$.

Ex. 4. Multiply $\frac{2}{5}$ by $\frac{3}{7}$ of $\frac{9}{11}$, or $\frac{2}{5}$ by $\frac{27}{77}$, Answer $\frac{54}{385}$.

Ex. 5. Multiply $9\frac{2}{5}$ by $7\frac{1}{3}$; or $\frac{47}{5}$ by $\frac{22}{3}$.

$$\begin{array}{r} 47 \\ 22 \\ \hline 94 \\ 94 \\ \hline 1034 \end{array} \qquad \begin{array}{r} 5 \\ 3 \\ \hline 15 \end{array}$$

$\frac{1034}{15}$ the Answer, which reduced to a mix'd Number is $68\frac{14}{15}$.

Ex. 6. Multiply $\frac{2}{33}$ by 7, or $\frac{2}{33}$ by $\frac{7}{1}$. Answer $\frac{14}{33}$.

Ex. 7. Multiply $4\frac{2}{5}$ by $\frac{3}{7}$, that is $\frac{22}{5}$ by $\frac{3}{7}$, produces $\frac{66}{35}$ which reduced, is $1\frac{31}{35}$.

Ex. 8. Multiply $\frac{2}{33}$ by 7, or $\frac{7}{1}$. Answer $\frac{14}{33}$.

But if a mix'd Number be to be multiplied by a whole Number, first multiply the fractional Part,

Part, and reduce it to its equivalent mix'd Number, ſubſcribing the fractional Part, but carrying the Integers to the Product of the Integers.

Ex. 9. Multiply $4\frac{2}{5}$ by 6.

Firſt, $\frac{2}{5}$ by 6 is $\frac{12}{5}$ which reduced, is $2\frac{2}{5}$, and 4 by 6 is 24, therefore the Product is $26\frac{2}{5}$.

CHAP. VI.

Division of vulgar Fractions.

The RULE.

REDUCE compound Fractions to ſingle ones, and mix'd Numbers to improper Fractions, and expreſs Integers in the form of a Fraction; then multiply the Numerator of the Dividend by the Denominator of the Diviſor, and the Product is the Numerator of the Quotient; and multiply the Denominator of the Dividend by the Numerator of the Diviſor, and the Product is the Denominator of the Quotient. But obſerve, that if the Numerators of the given Fractions are compoſed to one another, they may be divided by their greateſt common Meaſure; or if the Denominators are compoſed to one another, they

they may be divided by their greatest common Measure; and use these Quotes in their stead: And so the Fraction sought will be had in its least Terms.

Ex. 1. Divide $\frac{2}{7}$ by $\frac{3}{8}$.

$$\frac{3}{8}\Big)\,\frac{2}{7}\,\Big(\frac{16}{21}.$$

Ex. 2. Divide $\frac{24}{49}$ by $\frac{9}{14}$.

$$\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{14}}}\Big)\quad\frac{\overset{8}{\cancel{24}}}{\underset{7}{\cancel{49}}}\quad\Big(\frac{16}{21}$$

Ex. 3. Divide $\frac{1}{4}$ by $\frac{3}{5}$ of $\frac{5}{8}$, that is, when reduced $\frac{1}{4}$ by $\frac{3}{8}$.

$$\frac{3}{\underset{2}{\cancel{8}}}\Big)\quad\frac{1}{\underset{1}{\cancel{4}}}\quad\Big(\frac{2}{3}$$

Ex. 4. Divide $68\frac{14}{15}$ by $9\frac{2}{5}$, that is $\frac{1034}{15}$ by $\frac{47}{5}$.

$$\frac{47}{\underset{1}{\cancel{5}}}\Big)\quad\frac{1034}{\underset{3}{\cancel{15}}}\quad\Big(\frac{1034}{141}$$ Which reduced, is $7\frac{1}{3}$.

Ex. 5. Divide $\frac{14}{33}$ by 7, that is $\frac{14}{33}$ by $\frac{7}{1}$.

$$\frac{\overset{1}{\cancel{7}}}{1}\Big)\quad\frac{\overset{2}{\cancel{14}}}{33}\quad\Big(\frac{2}{33}$$

Ex. 5. Divide 9 by $\frac{2}{3}$.

$$\frac{2}{3}\Big)\,\frac{9}{1}\,\Big(\frac{27}{2}\text{ or }13\tfrac{1}{2}.$$

CHAP.

CHAP. VII.

The managing such Fractions *that have their Terms mix'd Numbers, or* Fractions. *The valuing any assigned Parts of any contract Number proposed, and its Converses. The Extraction of the Roots of* Fractions.

SECT. I.

To reduce a FRACTION *that hath its Terms mix'd Numbers or* FRACTIONS, *to another that hath its Terms Integers.*

The RULE.

ACCORDING to the Rules laid down in Division, divide the Numerator by the Denominator, and the Quotient is the Answer.

Ex. 1. Reduce $\frac{72\frac{12}{145}}{19\frac{7}{13}}$ to a Fraction, whose Terms are Integers.

$19\frac{7}{13}$) $72\frac{12}{145}$ (but these reduced, are $\frac{254}{13}$) $\frac{5952}{145}$ (which quotes $\frac{38688}{18415}$.

Ex. 2. Reduce the Terms of this Fraction $\frac{\frac{4}{7}}{\frac{2}{11}}$ to Integers.

$$\begin{matrix} 1 & & 2 \\ \frac{2}{11}) & & \frac{4}{7} \end{matrix} \left(\tfrac{22}{7}\right.$$ The Answer.

Ex. 3. Reduce $\frac{\frac{3}{4}}{7}$ to a Fraction whose Terms are Integers.

$$\frac{7}{1}) \ \frac{3}{4} \left(\tfrac{3}{28}\right.$$ The Answer.

Ex. 4. Reduce $\frac{3}{\frac{4}{7}}$ to a Fraction whose Terms are Integers.

$$\frac{4}{7}) \ \frac{3}{1} \left(\tfrac{21}{4}\right.$$ The Answer.

Ex. 5. Reduce $\dfrac{4\frac{2\frac{1}{2}}{5\frac{1}{3}}}{3\frac{1\frac{1}{3}}{5\frac{1}{2}}}$ to a Fraction whose Terms are Integers.

Which requires the dividing of $4\frac{2\frac{1}{2}}{5\frac{1}{3}}$ by $3\frac{1\frac{1}{3}}{5\frac{1}{2}}$

Where the fractional Part of the Divisor is $\frac{1\frac{1}{3}}{5\frac{1}{2}}$ which reduced according to the Rule is $\frac{8}{33}$; and

and the fractional Part of the Dividend is $\frac{2\frac{1}{2}}{5\frac{1}{3}}$ which reduced is $\frac{1}{3}\frac{5}{2}$: Therefore the Division now is $3\frac{2}{3\ 3}$) $4\frac{1}{3}\frac{5}{2}$(And so the Answer according to the Rules of Division is $\frac{4}{3}\frac{7}{4}\frac{1}{2}\frac{9}{4}$.

Hence it appears, how Fractions of howsoever complex a Nature, may be reduced to others of the most simple kind, *viz.* such as have their Terms Integers: And then they are fitted for the foregoing Rules.

SECT. II.

To value any fractional Part or Parts of any CONTRACT NUMBER *proposed.*

The RULE.

LET the proposed Number be reduced to the least Denomination therein express'd; then multiply it by the Numerator of the Fraction denoting the Parts, and divide the Product by the Denominator, and the Quotient, if nothing remain, is the Answer of the same Denomination, the contract Number was last reduced to: But if any thing remain, reduce it to the next lower Denomination, and then divide it by the same Divisor, and so the Quotient is another part of the Answer of the last Denomination, to be connected with the foregoing: And if there be still a Remainder, reduce and divide till you come to your desired Exactness: But lastly, if the Remainder's Name is the least that the contract Number is usually reduced to, and yet an exact Answer is sought for; make the Remainder the

H Numerator,

Numerator, and the Divisor the Denominator of a Fraction, to be annexed to the foregoing Value.

Example 1. What is $\frac{11}{24}$ of 7 C. wt. and 3 qrs?

```
       7
       4
      ---
      31
      11
     ----
24) 341(14
    101
      5
     28
    ----
24) 140(5
     20
```

Hence the Answer is 14 qr. 5 $\frac{20}{24}$ lb. But if the Quarters are reduced to Hundred Weights, and the Fraction to its least Terms, the Answer is 3 C.wt. 2 qrs. 5 $\frac{5}{6}$ lb.

Ex. 2. What is $\frac{22}{25}$ of 14 *s.* 5 $\frac{1}{2}$ *d.*?

```
       14  5 1/2
       12
      ----
      173
        2
      ----
      347
       22
      ----
      694
     694
    -----
25) 7634(305 9/25 half-pence, or
     134           s.  d.
       9           12  8 17/25.
```

If the Numerator of the Fraction proposed, be so small that the contract Number may be multiplied by it without Reduction: Then divide the higher part of the Product by the Denominator,

and the Quote is that part of the Anſwer that is of that Name; then reduce the Remainder to the next inferiour Name, and add thereto all that are of the ſame in the preceding Product, and divide this Sum by the ſame Diviſor, and the Quote is another part of the Anſwer, of the ſame Name with the laſt Dividend: Thus proceed to your loweſt Denomination, and if any thing ſtill remain, expreſs it as in the foregoing Operations.

Ex. 3. What is $\frac{7}{23}$ of 5 *l.* 7 *s.* 6 *d.*?

```
        l.  s.  d.
        5   7   6
                7    l.  s.  d.
23) 37  12  6  ( 1  12  8 14/23  The Answer.
    14
    20
23) 292
    62
    16
    12
23) 198
    14
```

But if the Numerator be not a ſingle Digit, yet if it be the Product of two Digits, multiply the contract Number by one of thoſe Digits, and the Product by the other, and this laſt Product is that made by multiplying the contract Number by the Numerator, which divide as in the laſt.

l. s. d.

Ex. 4. What is $\frac{42}{51}$ of 14 11 7 $\frac{1}{4}$?

$$
\begin{array}{rrrrl}
 & & & 6 & \\
\hline
 & 87 & 09 & 7\frac{1}{2} & \\
 & & & 7 & \quad l.\ s.\ d.\ q. \\
\hline
51) & 612 & 07 & 4\frac{1}{2} & (120\ 0\ 1\ 2\frac{48}{51} \\
 & 102 & & & \text{The Answer.} \\
 & 20 & & & \\
\hline
51) & 7 & & & \\
 & 12 & & & \\
\hline
51) & 88 & & & \\
 & 37 & & & \\
 & 4 & & & \\
\hline
51) & 150 & & & \\
 & 48 & & &
\end{array}
$$

Hence it appears convenient to reduce the Fraction to its least Terms, if it be not so already.

If the Numerator be not the Product of two single Digits, but differing from such a Product by a single Digit, first multiply the contract Number by that Product, and then the same contract Number by the Difference between the Product of the two Digits and the Numerator, and lastly, if the Numerator be greater than the Product of the two Digits, add, if less, substract the latter Product (arising from the contract Number) from the former; and then work as in the last.

Ex.

Ex. 5. What is $\frac{59}{101}$ of *l.* 37 *s.* 15 *d.* 03 $\frac{1}{4}$?

l.	*s.*	*d.*	
37	15	03 $\frac{1}{4}$	
264	06	10 $\frac{3}{4}$	The whole by 7
2114	15	02	That Product by 8, equal to the whole by 56.
113	05	09 $\frac{3}{4}$	The whole by 3.
101)2228	00	11 $\frac{3}{4}$	(*l.* 22 *s.* 01 *d.* 02 1 $\frac{50}{101}$.

208
6
20
120
19
12
239
37
4
151
50

Or thus,

l.	*s.*	*d.*	
37	15	03 $\frac{1}{4}$	
		63	Or 9 times 7
339	17	05 $\frac{1}{4}$	
2379	02	00 $\frac{3}{4}$	
151	01	01	
2228	00	11 $\frac{3}{4}$	

The Product as before.

But if the Numerator of the Fraction be a large Number, work according to the first Directions.

If instead of a contract Number, there had been proposed an abstract one, the Work might have been performed by this Section, or as the 3*d* Section of the 2*d* Chapter.

SECT.

SECT. III.

To find of what Whole any proposed CONTRACT NUMBER *is any proposed Part or Parts.*

The RULE.

Imagine the Numerator of the Fraction changed into its Denominator, and the Denominator into its Numerator, and then work as in the last Section.

		C.wt.	*qrs.*	*lb.*	*oz.*		
What is		03	02	05	$13\frac{1}{3}$	the $\frac{11}{24}$ of	
	C.wt.	*qrs.*	*lb.*	*oz.*			
	03	02	05	$13\frac{1}{3}$			
				24	Or 6 times 4.		
	21	01	07	00	*C.wt.*	*qrs.*	
11)	85	01	00	00	(7	3	The
	8						(Answer.
	4						
	33						

SECT. IV.

To find what Part or Parts any CONTRACT NUMBER *is of any other of the same kind.*

The RULE.

Reduce them both to the least Name found in either of the two, and then make the former the Numerator, and the latter the Denominator

minator of a Fraction, which reduce to its least Name ; and it shall show what Part or Parts the former is of the latter.

C.wt. qrs. lb. oz.
Ex. 1. What Part or Parts is 03 02 05 $13\frac{1}{3}$ of 7 *C.wt.* 3 *qrs.* ?

C.wt. qrs. lb. oz.	*C. qrs.*
3 02 05 $13\frac{1}{3}$	7 3
4	4
14	31
28	28
117	248
28	62
397	868
16	16
2395	5208
397	868
6365	13888
3	3
19096	41664

$\frac{19096}{41664}$.

41664	2
19096	5
3472	2
1736	
....	

$$1736)\ \frac{19096}{41664}\ (\tfrac{11}{24}$$

....
1736
19096
41664
6944
....

The Answer.

SECT.

SECT. V.

To extract the Root of a FRACTION.

The RULE.

FIrst reduce mix'd Numbers to improper Fractions, Compound ones to single ones, and the Fraction to its least Terms; then extract the Root of both Numerator and Denominator, and the Fraction formed by those Roots is the Root sought.

Example 1. Extract the square Root of $\frac{28}{63}$.

63	2
28	4
7	
•	

$7)\frac{28}{63}(\frac{4}{9}$ but the Root of 4 is 2, and the Root of 9 is 3; therefore the square Root of $\frac{28}{63}$ or its equal $\frac{4}{9}$, is $\frac{2}{3}$.

Extract the Square Root of $58\frac{7}{9}$.

This reduced is $\frac{529}{9}$, whose Square Root is $\frac{23}{3}$, equal to $7\frac{2}{3}$ the Answer.

I have omitted the Extraction, supposing the Reader already acquainted with the Extraction of the Roots of Integers; and because it comes not within the Scope of my Design.

If the Numerator and Denominator, when reduced to the least Terms, have not both the Root required, the Fraction cannot have that Root extracted: How to approach such Roots, you may see at the 14*th* Chapter, after it is reduced to a Decimal.

The

The Operation may be otherwiſe performed. For if either of the Terms be raiſed to the Power leſs by one, than is denoted by the Root, and then multiplied by the other Term, and then you extract out of that Product, the Root required of the Fraction ; I ſay this Root placed inſtead of that Term, which was multiplied into the Power of the other, forms the required Root of the given Fraction.

Example 1. What is the Cube Root of $\frac{16}{54}$?

$$\begin{array}{r} 16 \\ 16 \\ \hline 96 \\ 16 \\ \hline 256 \\ 54 \\ \hline 1024 \\ 1280 \\ \hline 13824 \end{array}$$

Whoſe Cube Root is 24, and therefore the required Root of the Fraction given is $\frac{16}{24}$, equal to $\frac{2}{3}$ if reduced to its loweſt Name. Or thus,

$$\begin{array}{r} 54 \\ 54 \\ \hline 216 \\ 270 \\ \hline 2916 \\ 16 \\ \hline 17496 \\ 2916 \\ \hline 46656 \end{array}$$

Whoſe Root is 36, and therefore the required Root of the Fraction is $\frac{36}{54}$ equal to $\frac{2}{3}$, as before.

Ex. 3. What is the Square Root of $\frac{28}{63}$?

$$\begin{array}{r} 28 \\ 63 \\ \hline 84 \\ 168 \\ \hline 1764 \end{array}$$

Whose Square Root is 42, and therefore the Square Root of $\frac{28}{63}$ is $\frac{42}{63}$, equal to $\frac{2}{3}$.

Or it is $\frac{28}{42}$, equal to $\frac{2}{3}$.

From whence it appears, that the Square Root of the Product of the Terms placed instead of either of those Terms, gives the Square Root of the Fraction proposed.

But lastly, observe, that if the required Root cannot be exactly had in such Product, the Fraction is a surd, and hath not the Root required.

CHAP.

CHAP. VIII.

Notation. Definitions. Theorems.

SECT. I.

NOTATION.

IN the first Section of the first Chapter, at the 11*th* Definition; a decimal Fraction was defined to be such a one, as had for its Denominator only an Unit prefixed to Cyphers: But if to the Numerator there be always prefixed so many Cyphers, that the Multitude of the Places in the Numerator be equal to the Multitude of Cyphers in the Denominator, then the Denominator will be always known; and therefore in the Notation is omitted, and the Numerator is written as a whole Number, only for Distinction, there is prefixed to it this Mark (.); And so $\frac{45}{100}$, $27\frac{137}{1000}$, $\frac{4}{1000}$, $24\frac{21}{1000}$, are written, .45; 27.137; .004; 24.21; And by .21; .125; .008; 9.012 is to be understood $\frac{21}{100}$, $\frac{125}{1000}$, $\frac{8}{1000}$, $9\frac{12}{1000}$. And by .1; .01; .001; .0001, *&c.* is to be un-

derſtood $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, &c. That is the Value of the Places below Unity, decreaſe in a decuple Proportion, as thoſe above Unity encreaſe.

From hence therefore it appears, that $\frac{1}{10}$, $\frac{11}{100}$, $\frac{111}{1000}$, $\frac{1111}{10000}$, are equal to .1; .11; .111; .1111; and theſe equal to $\frac{1}{10}$, $\frac{1}{10}$ and $\frac{1}{100}$, $\frac{1}{10}$ and $\frac{1}{100}$ and $\frac{1}{1000}$, $\frac{1}{10}$ and $\frac{1}{100}$ and $\frac{1}{1000}$ and $\frac{1}{10000}$.

And conſequently a Decimal may be read, by giving each place its due Denomination, or by reading it as a whole Number, and giving it the Name of the laſt place.

But the Notation of Decimals will appear from the Notation of Integers; for if the Mark of Diſtinction be placed to the Right hand of Unity, and then Figures continued, the firſt of theſe muſt be the tenth Part of what it would have been if it ſtood in the next place to the Left-hand (that of Unity) by the Value of Places, and ſo the firſt place below Unity gives Tenths, and are uſually called Primes.

And a Figure in the ſecond place below Unity, is but a tenth Part of what it would have been, if it ſtood in the firſt below Unity, for the ſame Reaſon; and ſo the ſecond place below Unity gives Tenths of Tenths, or Hundredths, and are uſually called Seconds. And ſo the reſt as in the following Table.

The

The Notation TABLE.

8	Tens of Millions.
6	Millions.
7	Hundreds of Thousands.
7	Tens of Thousands.
2	Thousands.
4	Hundreds.
8	Tens.
7	Units.
,4	Primes, or Tenth Parts.
3	Seconds, or Hundredth Parts.
2	Thirds, or Thousandth Parts.
7	Fourths, or ten Thousandth Parts.
6	Fifths, or hundred Thousandth Parts.
5	Sixths, or Millionth Parts.
7	Sevenths, or ten Millionth Parts, &c.

SECT. II.

DEFINITIONS.

24. A Terminate Decimal is that which runs downwards to a certain place, and there ends.

25. An Interminate Decimal is that which runs downwards continually, and no where ends.

So 476.5327 is terminate, but 17.4743 4743-4743 and so on continually, is interminate.

26. A compleat Decimal is either Terminate, or ſuch an Interminate as hath the ſame Figure or Figures continually Circulating; as 317.45 316 316 316 316 316, and ſo continually.

27. An approximate Decimal is ſuch, that hath ſome places true, but all the following ones uncertain.

So 327.645 + is called 327.645 more ſomething, and 21.5136— is called 21.5136 leſs ſomething, or which is the ſame thing, it is more than 21.5135, but leſs than 21.5136.

28. The Figure or Figures continually circulating, may be called a Repetend.

29. If one Figure only ſhould be continually annexed, call it a ſingle Repetend.

30. If more than one Figure continually circulate, call it a compound Repetend.

31. Like Repetends are ſuch as conſiſt of an equal Number of places.

32. Repetends that begin at the ſame place, whether at Units, Primes, Seconds, &c. and end at the ſame place, whether at fifths, ſixths, &c. are called conterminous.

34. Duodecimals are ſuch places of Figures below Unity that decreaſe in a twelfth Rate: And are called Primes, Seconds, Thirds, &c. and are marked by Daſhes, thus, 4176 7′ 11″ 7‴, &c, that is 4176 Integers, 7 Primes, 11 Seconds, 7 Thirds.

35. Sexageſimals are ſuch places of Figures below Unity, that decreaſe in a 60*th* rate; and are marked and diſtinguiſhed as in the foregoing.

SECT.

SECT. III.

THEOREMS.

11. IF the Mark of Distinction in any mix'd or Fractional Expression be moved one place towards the Left-hand, then every Figure, and consequently the whole Expression, is but a tenth Part of what it was before.

12. But if it had been moved towards the Right-hand, then every Digit, and so the whole, would have been ten times as much as it was before.

13. Cyphers to the Right-hand of Decimals, neither encrease nor diminish the Value of the Expression.

14. But Cyphers put between the separating Point and the Figures of a Decimal, diminish the Value of the Decimal.

15. Instead of .9999 and so on continually, put an Unit, for that is either equal to this, or else wants of it less than any thing assignable.

16. Any one of the Figures constituting a compound Repetend may be made the first thereof; for as in 4.23 175 175 175, &c. 175 is the Repetend after 4.23; so in 4.231 751 751 751, &c. 751 is the Repetend after 4.231.

17. Hence any two or more Repetends may be made to begin at the same place; for the Repetends of 4.235 235, &c. and 17.42 56 56 56 will begin at the same place, if the former begin at the thirds, and so its Repetend is 523, after 4.23, and that of the latter 56 after 17.42.

18. And any two or more Repetends may be made also to end together, and so to become conterminous, if they are continued so many places, that with the place where they begin together, the

the Number of them be the first common Multiple of the Numbers of each single Repetend: For the Number of places in 523 the Repetend after 4.23 (as in the foregoing) is 3, and the Number of places in 56 the Repetend after 17.42 is 2, but the least common Multiple of these is 6; therefore by continuing them 5 places below that where they begin together, they will be 4.23 5 23 5 23, &c. and 17.42 56 56 56, &c. that is conterminous; because after 4.23 the Repetend is 523523, and after 17.42 the Repetend is 565656.

19. Every Terminate may be consider'd as Interminate, by making Cyphers the Repetend.

20. In all results, if the Repetend consists of all Nines, reject them, and make the next superiour place an Unit more.

21. In all Results, if the Repetend consists of some other Repetend of fewer places, retain the latter only.

22. In all circulating Expressions dash the first and last of the Repetend, omitting the consequent places.

23. If a Divisor prime to its Dividend be 2 or 5, or any Power of either of them, or composed of any Powers of them, the Quotient will be Terminate: And the Cyphers to be annexed to the Dividend shall be as many as there are Units in the Index of the highest Power.

24. If a prime Divisor, no aliquot Part of the Dividend, be neither 2 nor 5, the Quotient shall be interminate; and the Cyphers to be annexed to the Dividend, in order to compleat the Repetend in the Quotient, shall be as many as the least Number of Nines written one after another, that is a Multiple of the Divisor, and that is the Number of the Places in the Repetend, which shall always be as many as the Units in the Divisor,

viſor, wanting one, or ſome aliquot part of the ſame.

25. The like is alſo true of Diviſors compoſed of Primes, neither 2 nor 5, if Prime to the Dividend.

26. If a Prime *b*, neither 2 nor 5, divide a Dividend no Multiple of *b*, and form in the Quotient a Repetend conſiſting of *c* Places; and alſo ſome Prime *d*, neither 2 nor 5, divide a Dividend, no Multiple of *d*, and form in the Quotient a Repetend conſiſting of *f* Places, then the Product of *b* and *d* dividing a Dividend Prime to it, ſhall form a Repetend conſiſting of as many Places as the Units in the leaſt common Multiple of *c* and *f*.

27. If the Power of ſome Prime *b*, neither 2 nor 5, dividing a Dividend Prime to it, form in the Quotient a Repetend of ſome Number of places *c*; then the next ſuperiour Power of the ſame Prime *b* dividing a Dividend Prime to it, forms a Repetend of a Number of Places expreſſed by *b* times *c*; except when *b* is 3, and the Index of its Power 1.

28. Any integral Diviſor prime to its integral Dividend given to find how many places the Repetend in the Quote conſiſts of: If the Diviſor have Cyphers in the right hand places, omit them; then if even divide by two as oft as poſſible; but if its right-hand place be 5, divide by 5 as oft as poſſible; and laſtly, divide Nines written one after another by the laſt Reſult till nothing remain; and the Number of Nines uſed ſhall ſhew the Number of Places in the Repetend. But note, that in dividing the firſt propoſed Dividend by its Diviſor, the Repetend ſhall begin after ſo many places in the Decimals are paſt, as the Cyphers cut off, and the Number of the Two's or Five's divided by.

29. If any Repetend once written as terminate be made the Numerator of a Vulgar Fraction, and its Denominator consists of as many places of Nines as the places of the Repetend, that Vulgar Fraction shall be equal to the Repetend, if written after the terminate part of the Expression.

30. If any required Root of some terminate Number be not exactly had from the Places given, it cannot be exactly had.

31. If any required Root of some circulating Expression doth not repeat from the Repetend once used, it cannot repeat at all.

CHAP. IX.

Addition of DECIMALS.

SECT. I.

Addition of TERMINATES.

The RULE.

PLACE like Places under like Places, imagine all Voids supplied with Cyphers, and then add as in Integers.

Add 476.2145 .903 4.17634 .00476325 3.146 together.

467.2145
.903
4.17634
.00476325
3.146

484.44460325 The Anſwer.

SECT. II.

To add INTERMINATES *that have ſingle Repetends.*

The RULE.

MAKE all conterminous, and then add as in the foregoing Section, only to the right-hand Column, add as many Units as there are Nines in it. And the Digit ſubſcribed to that Column ſhall be a Repetend.

Add $2.\dot{3}$ $2.\dot{7}$ $4.7\dot{6}$ $.\dot{3}$ 5.8 and $4.7\dot{3}$ together

$2.\dot{3}3$
$2.\dot{7}7$
$4.7\dot{6}$
$.\dot{3}3$
5.80
$4.7\dot{3}$

The Anſwer $20.7\dot{4}$

$47.2\dot{4}4$	
$2.\dot{8}88$	$2.7\dot{6}666$
$37.\dot{8}55$	$3.1\dot{8}555$
$.3.\dot{7}77$	$14.\dot{3}3333$
$38.47\dot{3}$	$21.7684.$
$48.76,$	$.7.3\dot{6}666$
$24.85.$	$24.\dot{8}8888$
$203.55\dot{0}$	$74.2795\dot{4}$

1769675	
.0020833	
358.8833333	
.003125 .	7.644
9764.9916666	2.838
.0010416	2.718
493.5958333	6 377
.0020833	.648
12387.1541666	20.222

SECT. III.

To add INTERMINATES *that have compound Repetends.*

The RULE.

MAKE all conterminous, and then add as in the first Section, only to the right-hand Column add as many Units as there are Tens in that Column where the Repetends all begin together; And then the Figures subscribed to the aforesaid Columns shall be the first and last of the Repetend.

34.7621621		
15.3141414	8.5935881	3.0459
4.1450641	2.9417653	.1763
54.2213677	11.5353535	3.2222

	2.2164	3117.087087
3.4127127	3.1463	623.541741
2.1414141	62.9262	1247.08348
6.5458731	314.6314	6.235417
12.0999999	382.9204	3871.572594
that is 12.1,		

37.42182	4.12432432
2.15	7.31212121
7.43222	6.45
47.00404	17.92644553

SECT. IV.

To add APPROXIMATES.

The RULE.

PLACE and work as in the first Section: Only observe that the certain Places of the Decimal are for the most part fewer by one than the decimal Places in any one of the Approximates given.

12.34518 −
10.5
.28
3.45455 −
1.41421 +
1.25928 +
29.2532 Certain.

This Rule is such, that we shall scarce ever err more than Unity in the last Place; but by two Operations, one made with more than just, the other made with less than just, we shall always be able to judge how far is certain, *viz.* as far as they agree.

12.34518	12.34517
10.5	10.5
.28	.28
3.45455	3.45454
1.41422	1.41421
1.25929	1.25928
29.25324	29.25320

Therefore

Therefore because they agree to four Places in the Decimals, so many are certain, as in the first Operation.

CHAP. X.

Substraction of Decimals.

SECT. I.

To Substract TERMINATES.

The RULE.

PLACE as in Addition, and imagine Voids supplied with Cyphers, and substract as in Integers.

483.525	217.45	314
47.178	184.9	211.435
436.347	32.55	102.565

217.5
116.875
100.625

SECT.

SECT. II.

To substract INTERMINATES *that have single Repetends.*

The RULE.

PLACE and continue as in Addition, and substract as in the first Section; only when the Repetend of the Subtrahend is greater than the Repetend of the Minuend, encrease the latter by 9, then substract, and carry one to the next place.

$27.64\dot{3}33$	$56.23\dot{6}$	24.700	12.76480
12.76460	12.200	$12.27\dot{3}$	$2.\dot{3}3333$
$14.8787\dot{3}$	$44.03\dot{6}$	$12.42\dot{6}$	$10.4314\dot{6}$

2.470	14.76438	$23.4\dot{3}$	$24\dot{3}.333$
$.24\dot{3}$	$8.5\dot{5}555$	14.77	125.274
$2.22\dot{6}$	$6.2088\dot{3}$	$8.6\dot{5}$	$118.05\dot{8}$

SECT. III.

To substract INTERMINATES *that have compound Repetends.*

The RULE.

PRepare as in Addition, and substract as in Integers; but if you borrow one, in substracting the place where both Repetends begin, add one

one to the Right-hand place of the Subtrahend. The Repetend in the Answer, is as in Addition.

48.7327327	31.71287	13.9778960
1.5171717	8.47272	8.7647647
47.2155610	23.23984	5.2131313

78.5222	48.7652000
3.7647	23.8548548
74.7874	24.9103451

SECT. IV.

To substract Approximates.

The RULE.

PLACE as in Addition, and substract as in the 1*st*. Section; and so the last place shall never err more than an Unit, if both are made more than just, or both less than just.

10.5	84.3275 +
3.45455 +	2.1847 +
7.04545	82.1328

CHAP. XI.

Multiplication of Decimals.

SECT. I.

To multiply TERMINATES.

The RULE.

PLACE and multiply as in Integers, and then from the Product to the Right-hand, cut off as many places for Fractions as there are fractional places in both Factors; but if it happen that there are not so many, supply the Defect by Cyphers to the Left-hand.

74.125	4.2346	.4251
671.3	.00125	.241
222375	211730	4251
74125	84692	17004
518875	42346	8502
444750	.005293250	.1024491
49760.1125		

SECT. II.

To multiply INTERMINATES *with single Repetends.*

The RULE.

IF the Multiplicand be Interminate, and the Multiplier a single Digit, multiply as in the foregoing Section, only to the last place of the Product, add as many Units as it contains Nines, and that place shall be a Repetend: But if the Multiplier be a Repetend, multiply by it as tho' it were a terminate Digit, placing the Product one place forwarder than ordinary towards the Left-hand; and divide the Result by 9, continuing the Quotient, till you arrive at a single or a compound Repetend; then beginning at the place under the last of the Multiplicand, cut off for Fractions, as taught in the first Section of this Chapter.

But if the multiplying Repetend be 3, imagine the Multiplicand carried one place towards the Left-hand, and then $\frac{1}{3}$ part of it, cutting off as before, shall be the Answer.

Lastly, If the Multiplier consists of many places, multiply by each place thereof, as taught above, and add the particular Products together, cutting of Fractions as above.

$48.7\acute{5}$	$3.17\acute{6}$	$845.\acute{3}$
4	$.\acute{3}$	$.07$
$195.0\acute{2}$	$.9530$	$59.17\acute{3}$

48.

48.734
.04
9)194936
2.16598

67.643
.06
9)405860
450985

26.458
.004
9)10582
11758024691 3

47.32
0.03
.15773

584.26
.3
194.755

721.48
.03
24.04851

748.64
.0634
299457
2245933
44918666
47.463057

487.65
2.174
9)195060
216733
341355
48765
97530
1060.36783

127647
4.13
425490
127647
510588
5276.0760

$$\begin{array}{r} 2.\dot{3} \\ 5.\dot{6} \\ \hline 9)140 \\ \hline 15\dot{5} \\ 11\dot{6}6 \\ \hline 13.\dot{2}2 \end{array} \qquad \begin{array}{r} 1.\dot{1} \\ 1.\dot{1} \\ \hline 9)1\dot{1} \\ \hline \dot{1}2345679\dot{0} \\ 1\dot{1}11111111 \\ \hline 1.\dot{2}345679 0\dot{1} \end{array}$$

$$\begin{array}{r} 48.75\dot{4} \\ 2.1\dot{3} \\ \hline 1625\dot{1}48 \\ 4875\dot{4}44 \\ 97508888 \\ \hline 104.0094\dot{8}\dot{1} \end{array}$$

$$\begin{array}{r} 3.4\dot{7} \\ 2.\dot{4} \\ \hline 9)1|39\dot{1} \\ \hline 15\dot{4}5679012\dot{3} \\ 69\dot{5}55555555 \\ \hline 8.5\dot{0}1234567\dot{9} \end{array}$$

SECT. III.

To multiply INTERMINATES *that have Compound Repetends.*

The RULES.

IF the Multiplicand be Interminate, and the Multiplier a single Digit; multiply as in the first Section of this Chapter; only add to the Right.

Right-hand place of the Product as many Units as there are Tens in that place which is formed by the Left-hand place of the Repetend : And the Repetend of the Product consists of places not more than the Repetend of the Multiplicand.

```
 58.7645        2184.6732
       7             .05
---------      ----------
411.35219      109.233662
```

But if the Multiplier consists of many places, multiply by each place as in the foregoing, and then add as taught in the 3*d* Section of the 9*th* Chapter.

```
      417.6325
         76.45
   -----------
      20881626
     167053013
    2505795195
   29234277277
   -----------
   31928.00712
```

```
     8.6714285
          1.19
    ----------
     780428571
     867142857
    8671428571
   -----------
   10.318999999
   that is 10.319
```

$$\begin{array}{r}
2.7\acute{4}74201\acute{} \\
7.03 \\
\hline
824226 0\acute{4} \\
192\acute{3}1941\acute{0}31 \\
\hline
193143\acute{}63636\acute{} \\
\text{that is } 19.314\acute{3}\acute{6}
\end{array}$$

$$\begin{array}{r}
1.4\acute{0}864197\acute{5}\acute{3} \\
80.1 \\
\hline
14\acute{0}86419 75\acute{3} \\
112\acute{6}913 58024 69 \\
\hline
11283\acute{2}22222 22\acute{2} \\
\text{that is } 112.83\acute{2}
\end{array}$$

But if the Multiplier be Interminate, ima gine the ſeparating Point of it moved toward the Left-hand, as many places as its Repetend conſiſts of, and then place it under the Multiplier given, and ſubſtract; and multiply the Multiplicand by the Remainder, as taught above.

Secondly, add the higheſt Figure in the Reſult to that which is as many places below it, as the Repetend of the Multiplier conſiſted of places; and then that next below the former to that next below the latter; and ſo on till you come to the loweſt place, if the Multiplicand be terminate; and the undermoſt Figures ſhall be the Product, whoſe Repetend ſhall be like that of the Multiplier. But if both Factors be interminate, then go on till you arrive at a Repetend, or a ſufficient degree of Exactneſs. But if by adding any Figure to ſome other below it, the added Figure is in-

increaſed; or if that to which it is added, and all the Figures between it and the Figure added, be Nines; then in either Caſe, the place to which there was added ſhall be one more.

The carefull Obſervance of the following Examples will ſufficiently explain the Rule.

This Example is here wrought at large, the better to inform the Reader; but the ſame may moſt conveniently ſtand thus, where all the added Figures are mentally added;

$$
\begin{array}{r}
48.76 \\
.1\cancel{3}4\cancel{5} \\
.0001 \\
\hline
1344 \\
\hline
19504 \\
19504 \\
14628 \\
4876 \\
\hline
6.553344 \\
6 \\
\hline
6.559344 \\
5 \\
\hline
6.559844 \\
5 \\
6.559894 \\
9 \\
\hline
6559\cancel{9}0\cancel{3}
\end{array}
\qquad
\begin{array}{r}
48.76 \\
.1\cancel{3}4\cancel{5} \\
.0001 \\
\hline
.1344 \\
\hline
19504 \\
19504 \\
14628 \\
4876 \\
\hline
6553344 \\
9893 \\
90 \\
\hline
6.559\cancel{9}0\cancel{3}
\end{array}
$$

If the Multiplier be a Repetend only, without any terminate part; there is nothing to be ſubſtracted.

```
   417.64
    4.217
 --------
   292348
   41764
  83528
 167056
 ----------
 1761.18788
      25391
      3640
 ----------
 1761.36401
```

```
        1433.531
         1.71990
   -------------
     1290177790
    129017779
    1433531
   10034717
   1433531
   -------------
   2465.52996690
         3132143
          2432
   -------------
   2465.53243243
```

that is 2465.5324

```
        2203.355
         28.5714
      ----------
         8813420
        2203355
      15423485
     11016775
    17626840
    4406710
    -------------
    62952.9370470
         3099999
           00000
    -------------
    62953.
```

```
    43222.105901
          .0185
    ------------
    21610529505
   34576847208
   4322105901
   -------------
   79.958959 1685
   80028987 0566
       3   9816
   -------------
   80.0389981666

     47253.375
         1.48
    ----------
     378027000
    189013500
    47253375
    ---------
    69934.995
    70094999
        05000
    ---------
    70005.
```

3142.1532
486.513
486

486.027

219950724
62843064
1885291920
251372256
125686128

1527171.2933364
8699882297
9933

1528699.9933297

3.145
4.297
.004

4.293

9436
283090
629090
1258818

13.5034363636363636
1694221694221
533 533

13.5169533

2172
11198

11

111.87

15209
173818
217272
2172727
21727272

243.063
206
3

243.306

But if the Repetend of the Product doth not ſo ſpeedily repeat as might be wiſhed for, proceed to a determined Exactneſs; as in the following Example, where I have continued the Product to 24 places in Decimals.

```
           21485.314
            481.7652
                   4
           ---------
            481.7648
         ------------
          1718828.14
          859412572
        12891188888
       150397200200
       214853143143
      17188281451451
      85941257257257
--------------------------------
10350868.153572772772772772772772
      78403340276123949995711668
         5 4451   72 40     8227
--------------------------------
10350878.504451277224049996822768
```

Laſtly, if it be deſired to find what is left out; obſerve that omitting as many of the laſt places as the Repetend of the Multiplier did conſiſt of, and the Product is 10350878.504451277224049996 which wants of the true Product $\frac{822768\frac{772}{999}}{999999}$ of an Unit in the laſt place, the place of 6 after the 3 Nines.

The Conſtruction of this Fraction is thus: Make the Repetend of the firſt Product 772 (beginning where the Work ends) the Numerator;

To which ſubſcribe for a Denominator as many places of Nines; To this prefix as many of the laſt places in the Anſwer as the places in the Multiplier's Repetend. Laſtly, to this Number ſubſcribe as many places of Nines for a Denominator.

But obſerve that if the Product be divided into Periods, beginning at the left Hand, each Period to conſiſt of as many places as the Repetend in the Multiplier did; and a Point put over the laſt place of each Period: Then the whole firſt Period may be added to the ſecond, and the Reſult to the third, and ſo on: But when the Sum of the Periods conſiſts of all Nines, or the added Period be increaſed an Unit; the laſt place in ſuch Sum ſhall be made one more; and ſo the true Product will be had.

To illuſtrate this obſerve the following Operation, which is the foregoing Example.

10350868.1535	727727	727727	&c. the 1ſt
103508	785043	512771	Product.
785043	512770	240498	
4	1	9	
	2		

10350878.5044512772240499, &c. true Prod.

But if the added Figures were omitted, which might be done without any Burthen to the Memory, the Operation would ſtand thus.

10350868.1535	727727	727727,	&c. the 1ſt
785043	512770	240498	Product.
4	1	9	
	2		

10350878.5044512772240499, &c. true Prod.

Hence

Hence it appears that from the first Product the true one may be obtained in one single Line, if we knew how the last Figure of each Period should be increased. In order to discover which, add the first Figure of the Period then to be added to the first Figures of the two succeeding Periods, and increase their Sum by the Carries from the Sum of the second Figures of the same Periods; and the Number of Tens herein contained, call the Carries from the three Figures.

Then in adding the first Period to the Second, encrease the last Place by the Carries from the 3 Figures.

But in adding any other Period to its following, when $\left\{\begin{matrix}0\\1\end{matrix}\right\}$ was carried to the last Period, if the three Figures make $\left\{\begin{matrix}10\\\text{not }20\end{matrix}\right\}$ increase the last place by $\left\{\begin{matrix}1\\0\end{matrix}\right\}$.

So in the foregoing Example, because the Sum of 1, 6, 7 the first Figures make 14, which carries 1; instead of adding 103508 the first Period to the Second, I add 103509 the First Period increased by 1; and then the Work stands thus;

$$\begin{array}{l}103508681535727727727727, \text{ \&c.}\\ \qquad 785044\end{array}$$

Secondly, because 7, 7, 7 make 21, and 1 was carried to the last Period; instead of adding 785044, I add 785045, and then the Work stands thus;

$$\begin{array}{l}103508681535727727727727, \text{ \&c.}\\ \qquad 785044512772\end{array}$$

Thirdly, because 5, 7, 7, with 0 the Carry of the Seconds, make but 19 not 20, and 1 was carried to the last place; I don't increase the Period

riod 512772 at all, but add it without Increase, and then the Work stands thus;

```
1035086815357277277277727, &c.
         785044512772240499, &c.
```

Lastly, if the Left-hand Figures of the first Product be brought down to the undermost Line, the Work will stand thus;

```
1035086815357277277277727, &c. first Product.
103508785044512772240499, &c. true Product.
```

SECT. IV.

To multiply APPROXIMATES.

The RULE.

WORK as in the first Section of this Chapter; but if both Factors are Approximate, the uncertain places of the Product shall be one more than the places in the longest Factor; but if one is compleat, the uncertain places shall be one more than the places in the compleat Factor.

```
  415.763+
   .12184−
  --------
   1663052
  3326104
  415763
 831526
415763
----------
50.65656392
```

But all the places below 50.6 are uncertain, by the Rule.

```
    3124.312
      .01452+
   ---------
     6248624
   15621560
  12497248
  3124312
 -----------
 45.36501024
```

But all the places below 45. are uncertain, by the Rule.

$$\begin{array}{r}
48.1765+ \\
2.16 \\
\hline
2890590 \\
481765 \\
963530 \\
\hline
104.061240
\end{array}$$

But all the places below 104.06 are uncertain, by the Rule.

But moſt of the Figures in the foregoing Examples of this Section are uſeleſs: Therefore, by the following Rule the uſeleſs places may be ſaved.

Having, according to the Rule, diſcovered how many decimal places will be in the Product, and alſo how many places will be uncertain; you may find the place of the Left-hand uncertain Figure; under which put the Units place of the Multiplier, and invert the Order of all the other places of it. Then in multiplying, begin at that Figure of the Multiplicand which ſtands over the Figure wherewith you are then a multiplying, ſetting down the firſt Figure of each particular Product directly underneath one another; yet herein you muſt have a due regard to the Increaſe which would ariſe out of the Figures to the Right hand of that Figure in the Multiplicand you then begin with; and in the Product the laſt place is uncertain.

To exemplify this, ſee the two firſt Examples worked over again.

```
          415.763+
         48121.0   Multiplier inverted.
     4157 by 1..4157
      415 by 2...831
       41 by 1.....41
        4 by 8.....33
Tens in 4 by 4......1
             50.63 but certain 50.6

      3124.312
     25410.0
      312..312 by 1
      124...31 by 4
       15....3 by 5
     45.1 but 45. only certain.
```

In the first of these Examples, the Fractions in both Factors are 8, and so many are the Decimal places in the Product; but the uncertain places are 7 by the Rule: Therefore the Left-hand uncertain place is the second in Decimals, under which I set the Units place of the Multiplier, and invert the Order of the rest, and then multiply as in the Example.

But when the Factors are both compleat, you may secure what place you please, by this Rule.

But the safest Way of securing all the certain places in a Product from Approximates; is by making a double Operation, *viz.* the one with more than just the Approximate Factors, the other with less; for then as far as these two Products agree, is certain.

And so the two first Examples of this Section will be thus.

415.763+	415.764–
12183+	12184–
1247289	1663056
3326104	3326112
415763	415764
831526	831528
415763	415764
50.6524062**9**	50.65668576

Whence it appears that 50.65 is certain, but no farther.

3124.312	3124.312
.01452+	.01453–
6248624	9372936
15621560	15621560
12497248	12497248
3124312	3124312
45.3650 1024	45.39625336

And ſo it appears that 45.3 is certain, but no farther.

So here it may be obſerv'd, that by this double Operation, we are certain of a Place more than by the former Rule; which will happen alſo in many other Examples.

CHAP. XII.

Division of DECIMALS.

SECT. I.

To divide by a TERMINATE.

The RULE.

DIVIDE as tho' all were Integers, but so place the separating Point in the Quotient, that the Sum of the Decimal Places in the Divisor and Quotient may be equal to those in the Dividend. But if the Decimal Places of the Divisor be more than those in the Dividend, add Cyphers as Decimals to the Dividend, till the Number of Decimal Places in the latter is at least equal to the Number of those in the former.

```
48.75)1567.45875(32.153
      10495
       7458
       25837
        14625
        ....
```

Here because there are 2 decimal places in the Divisor and 5, in the Dividend, I make 3 decimal places in the Quotient; for 2 in the Divisor with 3 in the Dividend makes 5 the Number of Decimals in the Dividend. If after Division there be

be a Remainder, imagine Cyphers as many as you please annexed to the Dividend as decimal places, which bring down, and proceed with in Division till you come to no Remainder, or a Repetend, or a sufficient Degree of Exactness.

```
25.6)74.375(2.9052734375
     2317
       1350
         700
         1880
           880
           1120
             960
             1920
              1280
               ...
```

Here the Decimals in the Dividend with the Cyphers annex'd are eleven; but in the Divisor only one: Therefore there are ten places of Decimals in the Quotient.

Divide 48173.6 by 2187 till you have ten Places of Decimals in the Quotient.

```
2187)48173.6(22.0272519433
      4433
        5960
       15860
         5510
        11360
           4250
          20630
            9470
             7220
              6590
                29
```

Divide 31645.7264 by 1.12 till there remain nothing, or till you find the Figures repeated.

```
1.12)31645.7264(28255.11285714
     924
      285
       617
        572
         126
          144
           320
            960
             640
              800
               160
                480
                 32
```

In this Example because there remain'd 32 when there was nothing but Cyphers to be brought down; and because the same Remainder 32 at last came again under the same Conditions; the next Quotient Figure, and also all the following will circulate, as is denoted in the Quotient by the dash'd Figures.

The several Varieties of valuing the Quote will clearly appear by the following Examples, where the Operations are omitted, because, according to the Rule, they are performed as in Integers.

```
   4217) 176118788(41764
   42.17) 176118788(4176400
 .04217) 176118788(4176400000
   4.217) 1761187.88(417640
  004217) 176.118788(41764
   42.17) 176.118788(4.1764
 .04217).0176118788(.041764
   4.217) .176118788(.041764
```

There

There is another very excellent Method for valuing the Quotient besides the foregoing; Thus,

Consider the Value of that Place in the Dividend under which the Units Place of the Divisor falls, in asking the first Question; for the same Value shall the first Figure in the Quotient always be of.

So, in the sixth of those Examples, where the Operations are omitted; the 2 in the Units Place of the Divisor falls under 6 the Units Place of the Dividend; therefore the first Place in the Quote gives Units.

In the last; the 4 in the Units Place of the Divisor falls under the 7 in the second place of Decimals of the Dividend; therefore the first significant Figure of the Quotient is in the second place of Decimals.

If the Divisor at any time consist of Figures with Cyphers annexed, such Cyphers may be omitted in the Operation, provided they be accounted for in valuing the Quotient.

```
24000)3176.48(.132353
        77
         56
          84
          128
            80
             8
```

Hence therefore to divide by 10 100 1000, *&c.* is but to move the separating Point 1 2 3, *&c.* places to the Left-hand.

SECT.

SECT. II.

To Divide by INTERMINATES *with Repetends.*

The RULE.

IMagine the separating Point of the Divisor moved as many Places to the Left-hand as the Divisor's Repetend consists of Places, and then substract it from the Divisor it self: And the separating Point of the Dividend moved towards the Left-hand as many Places as that of the Divisor, and then substracted from the Dividend. Lastly, divide with these Remainders, as taught in the first Section, and the Quotient shall be that sought.

```
16038.1) 73688943488 (4.594
  160.3   73688.9434
-----------------------------
15877.8) 72952.054054
         944085
         1501954
          729520
```

```
2.174) 1060.36783 (487.65
 .217   106.036783
-----------------------------
1.957)  954.33105
        17153
         14971
         12720
           9785
           ....
```

.008)

.008).23535802469123(26.47

0 .0235358024691

.008).2118222222222(

51

38

62

6

417.6325)31928.007212(76.45

.4176 21.928007

417.2149)31896.079105(

26910361

18774670

20860745

. . . .

.0317)395.273614(12456.73

.0000 .395273

.0317)394.878341(

778

1447

1798

2133

2314

951

. . .

We are to observe, that in substracting the Divisor or Dividend diminished from themselves, the Repetends are often omitted, because they are

are equal, and so would leave Cyphers in the Remainder ; as may be observed in the foregoing Examples.

$417.\cancel{6}3\cancel{2})709\cancel{3}3.6\cancel{4}336($

$.417 \qquad 70.9\cancel{3}36\cancel{4}$

$417.215)70862.7\cancel{0}97\cancel{2}(1\cancel{6}9.8\cancel{4}$

2914120

4108309

3533747

1960272

291412

In this Example the first and last Remainders are the same, and the same Figures are to be brought down from the Dividend, and therefore the same Figures must come over again in the Quotient.

If the Quotient Figures do not repeat so quick as may be wished for ; proceed only to your desired Exactness, and then value what is left out ; as in Multiplication, only let the Divisor be the Denominator.

$4\cancel{1}7.\cancel{6})2\cancel{1}3.47\cancel{6}134(.51117607\frac{\dot{2}222\frac{5}{9}\frac{8}{9}\frac{6}{9}\frac{2}{9}\frac{6}{9}}{4172}$

$.4 \qquad .2\cancel{1}347\cancel{6}$

$417.2)213.2\cancel{6}2658($

4662

4906

7345

31738

25346

31426

2222

If

If 58626 be written immediately after 2222, thus 222258626, and then 2222 substracted from it, the Remainder 222256404 make Numerator: To the Denominator 4172 annex as many Cyphers as there are Nines in the superiour Denominator, and from that substract 4172, and the Remainder 417195828 make Denominator, and the Fraction $\frac{222256404}{417195828}$ shall be equal to the foregoing.

SECT. III.

To divide APPROXIMATES.

The RULE.

Divide as in the first Section of this Chapter. But to know how many Places in the Quotient are certain; write the Divisor under that part of the Dividend which gives the first Figure in the Quotient; then count how many Places from the Left-hand Figure of the Divisor to the first doubtful place (whether in the Divisor or in the Dividend) for one less shall the certain Places of the Quotient be.

31.45275+)	21876.5437	8−(695.536
	30048937	
	1741462	8
	168825	30
	11561	550
	2125	7250
	238	5600

Here we stop, for by the Rule, no more places are certain; and also observe, that what is left out, is therefore uncertain too.

If after the Place in the Divisor, which limits the uncertain places, there be drawn a Line, as in the Example; all the Figures to the Right-hand of it may be omitted; and so the Work would then stand thus.

```
31.45275+)21876.54378—(695.536
            30048937
             1741462
              168825
               11561
                2125
                 238
```

When the highest uncertain place is a Nine, make that to the Left-hand of it, an Unit more, whether it be in Addition, Substraction, Multiplication, &c.

But if we make two Operations; *viz.* The one with a Divisor the nearest, more than just, and a Dividend the nearest, less than just; the other with a Divisor the nearest, less than just, and a Dividend the nearest more than just; then so many places, and no more are certain, as agree in the Quotient.

And so the Example will be thus.

31.45275+)21876.54378— (695.5367
31.45276—)21876.54377+ (695.5365

Therefore the Quotient certain is 695.536.

Hence it appears, that it is most sure to use a double Work; but observe that every compleat Decimal in such Operations, needs not be altered to a more or less than just.

CHAP.

CHAP. XIII.

Reduction of Decimals.

SECT. I.

To Reduce a Vulgar FRACTION *to a* DECIMAL.

The RULE.

DIVIDE the Numerator by its Denominator, and the Quotient will be the Answer.

Example I. $\frac{5}{7}$.

7)5.

.714285 The Answer.

Ex. II.

Example II. $\frac{53}{91}$.

```
91) 53.   (.582417  The Answer.
     750
      220
        380
         160
          690
           53
```

Example III. $\frac{23}{36}$.

```
36) 23.   (.638       The Answer.
     140
      320
       32
```

Example IV. $\frac{11}{640}$.

```
640) 11.  (.0171875 The Answer.
      460
       120
        560
         480
          320
          . . .
```

SECT.

SECT. II.

To Reduce MONEY, WEIGHT, *&c.* *to Decimals.*

The RULE.

FIRST express in a vulgar Fraction, what part or parts of the Integer, the given Money, *&c.* is, (as taught at *Sect.* 4. *Ch.* 7. of Part 1.) and then reduce that Fraction to a Decimal, and it will be that sought.

Reduce 4 *s.* 7$\frac{1}{2}$ *d.* to the decimal of a Pound Sterling.

4 7$\frac{1}{2}$	20		
12	12		960) 222. (.23125
55	240	$\frac{222}{960}$	300
4	4		120
222	960		240
			480
			...

This is the Substance of the Method usually proposed, to reduce Money, Weight, *&c.* to a Decimal. But the following one is much readier, and also every whit as easie and as general.

First, Reduce the least Denomination to the Decimal part or parts of the next Superiour, to the Right-hand of which prefix all that is given of that next superiour Denomination; *Secondly*, Reduce this mix'd Number to the decimal Part of the next superiour, to which prefix what is given of this last superiour, and so proceed till you arrive at the Decimal of the Integer sought.

Reduce

Reduce 7 *s.* 7 *d.* $\frac{1}{2}$ to the decimal Part of a Pound Sterling.

4	2.
12	7.5
20	7.625
	.38125

First, 2 Farthings divided by 4, quotes .5 *d.* and then 7.5 *d.* divided by 12 quotes .625 *s.* *Lastly*, 7.625 *s.* divided by 20, quotes .38125 *l.*

But if any Divisor greater than of 12, be composed of two Digits, divide by those Digits continually, and you have the Quotient required; So in the following Example, to divide by 28; I first divide by 4, and the Quotient thence arising by 7.

Reduce 9 *oz.* 17 *d. wt.* 14 *gr.* to the Decimal of a *lb.* Troy.

Instead of 24 {	2	14
	12	7
	20	17.583
	12	9.87916
		.8232638

The Answer.

Reduce 3 *Cwt.* 3 *qrs.* 3 *lb.* to the Decimal of a Tun.

Instead of 28 {	4	3.
	7	.75
	4	3.10714285
	20	3.7767857142
		1 88839285714

The Answer.

Reduce

Reduce 41 Gallons and 5 Pints to the Decimal of a Hogshead, containing 63 Gallons.

	8	5.
Instead of 63	9	41.625
	7	4.625
		.660714285

The Answer.

Reduce $8'\ 10''\ 11'''$ a Duodecimal to a Decimal.

12	11.
12	10.916
12	8.90972
	.742476851

The Answer.

Reduce $42'\ 11''\ 53'''$ a Sexagesimal to a Decimal.

60	53.
60	11.883
60	42.19805
	.703300925

The Answer.

But the Decimal of any Number of Shillings, Pence and Farthings, as 6 *s.* 9 *d.* $\frac{1}{2}$ may be written down in one Line without any Burthen to the Memory: For if in the place of Primes be written half the Number of Shillings (.3), and the Pence reduced to Farthings (38) written after them if less than 6 *d.* or written after them encreased by one, when as much as, or more than 6 *d.* then

then this (.339) ſhall be the Decimal true to three Places; And if (7) half the Number of Farthings rejecting Sixpences, be divided by 12, the Quotient (58ʒ) written after the three places (.339) ſhall compleat the Decimal (.33958ʒ) of 6 *s.* 9 *d.* $\frac{1}{2}$.

But obſerve that if the Number of Shillings be odd, then the ſecond place of Decimals ſhall be encreaſed by 5; and ſo the Decimal of 7 *s.* 9 *d.* $\frac{1}{2}$ is .38958ʒ.

And the like of any other Sum, as 17 *s.* 10 *d.* $\frac{1}{4}$; for the half of 17 is 8.5 and the Number of Farthings 41 greater than 6 *d*, and therefore the 3 firſt Places are .892; and the Farthings rejecting Sixpences are 17, whoſe half 8.5 divided by 12 quotes 708ʒ, and this annexed to .892 gives the Decimal ſought .892708ʒ.

There are alſo particular Methods for expreſſing Weights, Meaſures, &c. Decimally, without waſte Figures or Burthen to the Memory; which for Brevity's ſake I omit.

Only obſerve that .00274 (becauſe very near the Decimal Part of a Year, anſwering to one Day) multiplied by the Number denoting any Days, gives the Decimal Part of a Year, anſwering to thoſe Days very near. And this becauſe of its uſe in Intereſt and Annuities, I would not omit.

SECT. III.

To reduce a Decimal to the leaſt Vulgar poſſible.

The RULE.

MAKE the Decimal the Numerator of a Fraction, whoſe Denominator ſhall be an Unit with as many Cyphers annexed as there are places

places in the Decimal given; and then if the Numerator be Interminate, diminish it as an Interminate Divisor in Division, and the Denominator as a Dividend to it; And lastly reduce it to its least Terms, and it shall be that required.

Reduce .6875 to the least vulgar Fraction possible.

$\frac{6875}{10000}$

10000	1	625)10000(16
6875	2	3750
3125	5	0000
625		
...		625) 6875 (11
		625
		... Answer $\frac{11}{16}$.

Reduce .74247685x to the least vulgar Fraction possible.

$$\frac{74247685x.}{1000000000}$$

$$\begin{array}{r} 74247685x \\ 742476 \\ \hline 741734375 \end{array}$$

$$\begin{array}{r} 1000000000 \\ 1000000 \\ \hline 999000000 \end{array}$$

$$\frac{741734375}{999000000}$$

999000000	1
741734375	2
257265625	1
227203125	7
30062500	1
16765625	1
13296875	3
3468750	1
2890625	5
578125	
......	

578125) 999000000 (1728
4208750
1618750
4625000
....

578125) 741734375 (1283
1636093
4798437
1734375
......
..

Answer $\frac{1283}{1728}$.

SECT.

SECT. IV.

To value any DECIMAL *in a lower Denomination.*

The RULE.

MUltiply the given Decimal by the Number of the leſſer Denomination contained in one of the greater, and the Product ſhall be the Anſwer.

Reduce .317 *l.* to Shillings.

$$\begin{array}{r} .317 \\ 20 \\ \hline 6.340 \end{array}$$ Shillings.

And ſo the Anſwer is 6.34 *s.* But if it had been required to value .317 *l.* in Shillings, Pence and Farthings; Firſt value it in Shillings, and then value the Decimal Parts of Shillings in Pence, and if there be any Parts of Pence value them in Farthings.

Value .317 *l.* in Shillings, Pence and Farthings.

$$\begin{array}{r} 317 \\ 20 \\ \hline 6.340 \\ 12 \\ \hline 4.08 \\ 4 \\ \hline .32 \end{array}$$ Anſw. 6 *s.* 4 *d.* 0.32 *f.*

And in the ſame manner value Weights and Meaſures.

Value .3928571$\dot{4}$ *Tun* in *Cwt. qrs.* & lb.

.3928571$\dot{4}$
20
―――――
7.857142$\dot{8}$
4
―――――
3.42857$\dot{1}$
28
―――――
342857$\dot{1}$
857142$\dot{8}$
―――――
11.99999$\dot{9}$

Anſwer 7 *Cwt.* 3 *qrs.* 12 lb.

For by the 5th Theorem a Repetend of Nines is an Unit more in the next ſuperiour place.

Value .8230902$\dot{7}$ lb *Troy*, in Ounces, Penny-weights and Grains.

.8230902$\dot{7}$
12
―――――
9.877083$\dot{3}$
20
―――――
17.5416$\dot{6}$
24
―――――
2166$\dot{6}$
1083$\dot{3}$
―――――
13.0000

Anſwer 9 *oz.* 17 *dwt.* 13 *gr.*

But

But the Decimal Part of a Pound Sterling may be valued ſhorter, thus. Subſtract all the Primes, and a half Prime if there be one; then multiply the Remainder by 4, placing the Product under the Remainder but two places towards the Right hand, which taken from the aforeſaid Remainder, leaves a Number, from which cut off to the Right-hand, as many places as there were in the firſt Remainder, by a ſeparating Point; and the double of the Primes and half Primes give the Shillings, and the Figures to the Left-hand of the ſeparating Point, give the Farthings, which reduce to Pence and Farthings, and thoſe to the Right-hand give the Decimal of a Farthing.

Value .3647 *l.* in Shillings, Pence and Farthings.

```
.3647
.35
-----
 147
   588   That is 147 by 4.
-----
 14.112
```

Anſwer 7 *s.* 3 *d.* 2.112 *f.*

But the Primes and the half Prime when there is any may be conceived to be ſubſtracted, and the Product made of the Remainder by 4 may be ſubſtracted, without ſetting down as in Diviſion. And then the true Value of any Decimal Parts of a Pound Sterling are had without any waſte Figures, or Burthen to the Memory.

Value .3741 *l.* in Shillings.

Anſwer 7 *s.* 5 *d.* 3.136 *f.*

But if the Shillings be valued, as before, and the remaining Figures in the ſecond and third places, abating one in 25, be called Farthings; the Value of the Decimal ſhall be had ſo near, that the Error ſhall never come to a Farthing.

CHAP.

CHAP. XIV.

Extraction of Roots of Decimals.

The RULE.

SET a Point over the Place of Units, and also over every Place upwards and downwards as taught in Integers; and then extract as in Integers.

What is the Square Root of 272.25?

```
 .  .  .
 2 7 2.2 5 | 1 6.5
 ----------|------
   8 6 1 2 5 | 3 2 5
   8 1 2     |
 . . . .     |
```

Answer 16.5

For the Places of Integers in the Root shall be as many as there are Points over the Integers.

If the Number given have not an exact Root, by bringing down a Period of Cyphers every time, you may proceed to a sufficient Exactness.

If at any time there are Figures to be brought down, but they are not a Period (that is 2 in the Square, 3 in the Cube, &c.) supply the Defect with Cyphers.

Ex-

Extract the Square Root of .04173.

.04173 | .2042792
865 | .21355
·5700 | 41
161800
·1883550
4507950
422368

Answer .2042792, &c.

Extract the Cube Root of .0003086964798.

.000308696479800000 | .067584
92696 | 36
387 108.. | 889
1309 | 4489
12109 | 6725
7933479 | 455625
2015 13467.. | 108064
10075 | 45670564
1356775
1149604800
20258 1366875..
162064
136849564
54808288000
202744 137011692..
810976
13701980176
367296

Answer .067584, &c.

CHAP.

CHAP. XV.

Approximations to FRACTIONS, *or Proportions in smaller Terms.*

LET the Fraction $\frac{2684769}{8376571}$ (or the Proportion of 2684769 to 8376571) be assigned, to find one equal to it, if possible, or at least the next greater, or the next lesser, which may be expressed in Numbers not greater than 999; that is in Numbers not exceeding 3 Places.

If the Fraction sought (whose Terms are not to be greater than [999] a given Number) be the next greater than a Fraction proposed, divide the proposed Fraction's Denominator [8376571] by its Numerator [2684769]; (if the next lesser divide the Numerator by the Denominator) continuing the Quotient [3.12003416] in Decimal Parts to such an Accuracy as shall be sufficient; which

which Quotient [3.12003416] for the next greater, is to be the Denominator anſwering to the Numerator 1: But for the next leſſer, it is to be the Numerator anſwering to the Denominator 1, compleating a Fraction as near as ſhall be neceſſary to that propoſed, which Fraction [$\frac{1}{3.12003416}$] I call the firſt Fraction compleat: And the ſame wanting the Appendage of Decimal Parts, I call, [$\frac{1}{3}$] the firſt Fraction curtailed.

Then by this Appendage [.12003416] of the firſt Fraction, Divide 1 Integer, and by [8] the Integer Number which is next leſs than the full Quotient [8.+], Multiply both Terms of the firſt Fraction compleat; and the Products [8 & 24.96027328] of ſuch Multiplication, I call the continual Increments of thoſe Terms reſpectively. And [.03972672] ſo much as the Appendage of Decimal Parts in ſuch continual Increment wants of 1 Integer, I call the Complement of the Appendage of the continual Increment.

Then both to the Numerator [1] and the Denominator [3.12003416] of the firſt Fraction, add (reſpectively) its continual Increment, which make [9 & 28.08030744] the Terms of the ſecond Fraction, and theſe again (reſpectively) increaſed by the ſame continual Increments, make the Terms of the third Fraction [17 & 53.04058072]; and ſo onward, till the Fraction [$\frac{25}{78.00085400}$] ſo ariſing, hath an Appendage, leſs than the Complement of the Appendage of the continual Increment.

And then that Fraction I call the laſt of the firſt Order; which alſo is to be the firſt of the ſecond Order.

By the Appendage of this Fraction (the firſt of the ſecond Order), divide the Complement of the Appendage of the continual Increment of the

foregoing Order, and by [46] the Integer Number next leſs than [46.+] the full Quotient of ſuch Diviſion, Multiply each Term of the firſt Fraction of this Order; and to [1150 & 3588.-03928400] the Products (reſpectively) add the continual Increments of the foregoing Order [8 & 24.96027328]; the Reſults of which [1158 & 3612.99955728], I call the Increments of the preſent Order. And [.00044272] ſo much as the Appendage of ſuch continual Increment wants of 1 Integer, I call (as before) the Complement thereof.

Then to [25 & 78.00085400] the Terms of the firſt Fraction of this (ſecond) Order, add the reſpective continual Increments of this ſame Order, and ſo continually; for the ſecond, third, and ſubſequent Fractions of this Order, ſo long as there is an Appendage as great as the Complement of the Appendage of the continual Increment of the ſame Order; and when the Appendage is leſs, ſuch Fraction is laſt of the preſent Order, and firſt of the following.

[In this Example the Terms of the ſecond Fraction of this Order, are 1183 & 3691.00041128; Therefore this Order hath no third Fraction, for the Appendage is leſs than the Complement of the Appendage of the continual Increment. We therefore ſhould proceed to the third and ſubſequent Orders, but that the Terms of the Fraction laſt found $\frac{1183}{3691}$ are greater than [999] the propoſed Limit; and becauſe after $\frac{25}{78}$ there is none (before this) nearer to the juſt Value, it is evident that $\frac{25}{78}$ is the Fraction ſought, as being neareſt (to the true Value) of any (greater than it) in Terms not greater than 999, and alſo in the ſmalleſt Terms.]

But

But when the Limits will permit it, we may proceed, making up (as is already ſhewn) the continual Increments of each Order, of ſuch Multiples of the Terms of the firſt Fraction of the ſame Order, adding thereunto the continual Increments of the Order next aforegoing; and continuing each Order ſo lo long as there is a ſufficient Appendage, (not leſs than the Complement of the Appendage of the continual Increment of the ſame Order;) And when the Appendage becomes leſs than ſuch, this Appendage dividing that Complement, ſhews by the Quotient (that is, by the greateſt Integer Number leſs than it) how many times the Terms of that Fraction (where this happens) are to be taken, together with the continual Increments next foregoing, to make the continual Increments of the ſucceeding Order.

And the Fractions thus ariſing (which I call the firſt, ſecond, third, *&c.* of the firſt, ſecond, third, *&c.* Order) without their Appendages of Decimal Parts, (which therefore I call Fractions curtailed) do continually more and more approach to the Value of the Fraction propoſed; and are each of them the neareſt greateſt, or the neareſt leſſer (as is ſaid) of any not conſiſting of greater Terms: Nor is there (in Integers) any other ſuch intermediate Approaches. Of all which, if we make Choice of ſuch as have the greateſt Terms, not exceeding the Limit propoſed, we have what was required.

And what is ſaid of the Numerator and Denominator of Fractions (whether proper or improper) is eaſily applicable to the Antecedent and conſequent Terms of a Proportion.

But here it is fit to be noted, that in ſeeking the firſt Fractions (by Diviſion it is convenient to continue the Quotient to at leaſt twice as many Places (or ſomewhat more) of Decimal Parts, as

Q 2 are

are the Places of that Number proposed as the Limit, greater than which the Terms of the Fraction sought are not to be.

Among the Multitude of Examples of the like Process, if it be required to find Integers that will express the next least, or next greater Proportion than that of 1 to 3.14159265358979 3+ (which is that of the Diameter of a Circle to its Circumference) under a given Limit; We may not only speedily attain the famous Numbers of *Archimedes* 7 & 22, and also those of *Metius* 113 & 355, (the Invention of which hath raised Wonder in Ingenious Men, and rendered the Memory of the Authors Immortal) but also discover others much nearer in Integers, though in higher Numbers. And he that is unwilling to take the Pains to do it himself, may find it ready done to his Hands; and also this Subject largely enquired into, by the Reverend Dr. *Wallis*, in the 10*th* and 11*th* Chapters of his History of *Algebra*: From whence I extracted this Chapter.

CHAP.

CHAP. XVI.

Of DUODECIMALS.

SINCE Duodecimals are the Foundations of many of our *English* Meaſures, to ſay ſomething of the Management of them may be very uſeful, eſpecially to the Builder. But becauſe Addition and Subſtraction, are performed in all reſpects like Addition and Subſtraction of Pence, I ſhall omit their Rules and Operations. And becauſe Multiplication is the very neceſſary Part, but Diviſion only when the Diviſor is an Integer, and Extractions ſcarce at all, when we meaſure according to any of the various Cuſtoms uſed: Beſides, if ſuch Operations were required, it is better to reduce them to Decimals; I ſhall therefore treat of Multiplication fully, of Diviſion ſparingly, and omit Extractions and Reductions.

To multiply Integers, Primes, Seconds, *&c.* by Integers, Primes, Seconds, *&c.*

The RULE.

SET the Multiplier so under the Multiplicand, that the Integers of the Multiplier stand under the lowest Place of the Multiplicand, and the other part of the Multiplier in order, to the Right-hand; then multiply the Multiplicand by each Term of the Multiplier, beginning at the Right-hand, setting the Right-hand place of each Line under the Multiplying Figure, and carrying one for every 12 to the next place.

Ex. 1. Multiply 5 03′ 03″ by 3 05′ 07″.

```
       '  "
     5 03 03
          3 05 07
     -------------
       03 00 1  09
     2 02 04 03
    15 09 09
    --------------
    18 03 02 01 09   The Answer.
```

Ex. 2. Multiply 6789 06′ 04″ by 24 03′ 08″.

```
            '  "
      6789 06 04
              24 03 08
      ------------------
       377 04 04 02 08
      1697 04 07 00
    162948 06 00
    -------------------
    165023 02 11 02 08   The Answer.
```

In

In the 2*d* Example after I had multiplied $05'$ $04''$ by $08''$ and stated $02'''$ $08''''$ for the Product, besides $04''$ to be carried, I multiplyed 6789 by $08''$, which with $04''$ carried, produced $54316''$ which reduced according to the common Rules of Reduction, becomes 377 $04'$ $04''$; And in multiplying by $03'$ I proceeded in like manner when I came to the Integers of the Multiplicand; which being duly observed, sufficiently illustrates the Rule.

But besides the foregoing general Method, there are many particular ones adapted to particular Cases, which render the Operations very easie; and the most Remarkable follow.

To multiply any Integers, Primes, Seconds, &c. by 12 Integers; first carry the Duodecimals one place higher, till you come to Integers, and then to the Product of the Integers of the Multiplicand by 12, add the Primes of the Multiplicand.

Ex. $417\ \ 07'\ \ 09''\ \ 11'''$ by 12.

$5011\ \ 09\ \ 11$

To multiply any Integers, Primes, Seconds, &c. by a Multiple of 12 Integers; first Multiply by 12, and then the Product by the Number expressing what Multiple the Multiplier is of 12.

$417\ \ 07'\ \ 09''\ \ 11'''$ by 72, or 6 times 12.

$5011\ \ 09\ \ 11$

$30070\ \ 11\ \ 06$ The Answer.

But

But if the Multiplying Integer consists of some Multiple of 12, and some Number over; first multiply by the Multiple, and to the Product add that made by the Multiplicand, multiplied by the Excess of the Multiplier above the Multiple.

	′	″	‴		
417	07	09	11	by 77, or 6 times 12 with 5.	
5011	09	11		12	times the Multiplicand.
30070	11	06		72	
2088	03	01	07	5	
32159	02	07	07	77	

To multiply Integers, Primes, Seconds, &c. by any Number of Primes that is an Aliquot Part of 12: Take such a Part of the Multiplicand, as the Multiplier is of 12′, and if any thing remain, either in Integers, Primes, &c. esteem it so many such Parts of that Place, but value it in a Number of the next Place. So in the following Example, after the 3*d* Part of 317 is taken, there remains 2, which are Thirds of an Integer, and because $\frac{1}{3}$ of an Integer is 4′, and so $\frac{2}{3}$ are 8′, I carry 8′ to 2′ the Third Part 7′, which makes 10′; and so in any other Place.

			′
317	07	05	by 4
105	10	05	08 The Answer.

But if the Multiplier be 1, take a twelfth Part of the Integers, and place the Remainder under Primes, and move the Primes, Seconds, &c.

&c. in the Multiplicand one place lower in the Product.

317 07′ 05″ by 01′

26 05 07 05 The Answer.

To multiply any Number of Integers, Primes, Seconds, &c. by a Number of Primes that is an Aliquant Part of 12′; first see what aliquot Parts of 12′ added together, will make the Multiplier; then multiply the Multiplicand by such Aliquot Parts, and the Sum of the Products shall be the Answer.

317 07′ 05″ by 10′ or 6′ and 4′.

158 09 08 06
105 10 05 08

264 08 02 02 The Answer.

172 07′ 06″ by 11′ or 4′ & 4′ & 3′.

57 06 06
57 06 06
43 01 10 06

158 02 10 06 The Answer.

But in this Case, the Answer may be had, by taking a twelfth Part of the Multiplicand from the Multiplicand.

172 07′ 06″ by 11′

14 04 07 06 Equal to the 12*th* Part of the Multiplicand.

158 02 10 06 The Answer.

To multiply Integers, Primes, Seconds, &c. by Seconds or Thirds, &c. multiply as for Primes, only set the Product one place lower for Seconds, two for Thirds, &c. But observe that if in such Operations, the Primes are more than 11 they shall be reduced to Integers, as in the Second Example of this Chapter; and the like of any other Duodecimal place.

```
     '   ''       ''
767 03  05   by   3
191 09
15 11 09  10 03
```

And all these Directions put together, shew us many easie and expedient Methods for multiplying Integers and Duodecimals by Integers and Duodecimals; and are, as I conceive, the most material.

To divide Integers, Primes, Seconds, &c. by an Integer, first divide the Integers; and if any thing remain, reduce it to Primes, to which add the Primes given, and then divide for Primes; and if any thing remain, reduce it to Seconds, and work as before.

```
                          '  ''  '''
45) 732. 04 08 (16 03 03 07 33/45 or 11/15.
    282
     12
   )148)
     13
   )164(
     29
   )348(
     33
```

What-

Whatſoever hath been ſaid of Decimals concerning Interminates, whether certain or uncertain, may be applied to Duodecimals or Sexageſimals; if what is there ſaid of 9, be here applied to 11 and 59.

Having entered on the handling of Duodecimals, whoſe uſe is in Meaſuring, it may not be amiſs to conclude with an Idea of Meaſuring, different from what is taught by our Modern Authors, and what may help to correct the falſe Ideas ſo common of Multiplication, even in ſome of our beſt Authors, and conſequently not improper in a Treatiſe of Numbers.

CHAP. XVII.

Of MEASURING.

EVERY Magnitude is meaſured by ſome Magnitude of the ſame kind. A Line by a lineal Foot, Yard, &c. A Superficies by a Square Foot, Yard, &c. A Solid by a Cubick Foot, Yard, &c.

The Lineal Meaſure is known to all.

The Superficial Meaſure may be conceived, by imagining a Floor paved with Tiles, each a Square Foot; for then the Number of Tiles is equal to the Number of Square Feet in that Flooring. Now if the Flooring be juſt one Foot broad, the Number of Tiles (or of Square Feet) will be equal to the Number of Lineal Feet, in the Length of the Floor: But if the Flooring be 2, 3, 4, 5, &c. Feet broad, the Number of Tiles (or of Square Feet) will be twice, thrice, four-times, five-times, &c. ſo many Tiles (or Square Feet.) So if the Floor were 11 Foot long, and 7 Foot broad, 7 times 11 Tiles (or Square Feet)

Feet) gives 77 the Number of Tiles (or Square Feet) in that Flooring.

The solid Measure may be conceived by imagining a Wall built with Stones each a Cubick Foot; for then the Number of Stones will be equal to the Number of Cubick Feet in that Wall. First therefore if the Wall be one Foot thick, and one Foot high, the Number of Stones (or Cubick Feet) will be equal to the Number of Lineal Feet in the Length of the Wall. *Secondly*, If the Wall should be of the same Length and Heighth one Foot, as before, but the thickness 2, 3, 4, 5, *&c.* Feet (instead of one Foot); then the Number of Stones (or Cubick Feet) will be accordingly twice, thrice, four times, five-times, *&c.* as many as before. *Lastly*, if the Length and thickness be the same as in the last supposition, but the Heighth (instead of one Foot) be 2, 3, 4, 5, *&c.* Feet; the Number of Stones (or Cubick Feet) will be accordingly twice, thrice, four-times, five-times, *&c.* what it was in the foregoing. So if a Wall is 7 Foot long, 3 Foot thick, and 5 Foot high. From what hath been said, a Wall of 7 Foot long, 1 Foot thick, and 1 Foot high, consists of 7 Cubick Feet; but a Wall of 7 Foot long, 3 Foot thick, and one Foot high consists of 3 times 7 CubickFeet, that is 21 Cubick Feet; *lastly*, a Wall (of 7 Foot long and 3 Foot thick) as before, but 5 Foot high contains 5 times as many, that is 5 times 21 Cubick Feet, or 105 Cubick Feet.

From all which, its is evident, that in casting up any mensuration, the Multiplier in any of the Multiplications is an abstract Number, as well as in all other Multiplications whatsoever. Which may prevent the false Consequences usually drawn from multiplying Feet by Feet, *viz.* That of multiplying by a contract Number; as

3 *l.*

3 *l.* 19 *s.* by 3 *l.* 19 *s.* or half a Crown by half a Crown; which is contrary to the Nature of Multiplication, whose Operations are only Compendious Additions, either of the Multiplicand or some part of it, continually to itself, or its part.

FINIS.

ERRATA.

PAGE 2. Line 20. dele *and all Numbers less than* 10000, *that are not in the following Table are composed.*

Pag.	Lin.	For	Read
15	4	$\frac{3}{1}\frac{1}{1}\frac{8}{1}\frac{7}{1}\|^{3}$	$\frac{3}{1}\frac{1}{1}\frac{8}{1}\frac{7}{1}\|^{2}$
22	3	*ibid.*	*id.*
34	18	*improper*	*Compound*
48	3	*quotes* $\frac{3}{1}\frac{8}{8}\frac{6}{4}\frac{8}{1}\frac{8}{5}$	*quotes* $\frac{6}{1}\frac{7}{8}\frac{9}{4}\frac{3}{1}\frac{8}{5}$
59	6	11 *Definition*	17 *Definition*
59	21	24.21	24.021
67	2	467.2145	476.2145
ib.	20	473	4.73
68	1	1769675	1769.675
69	5	17.926445533	17.886445533
75	9	117580246913	.117580246913
75	11	.15773	1.5773
75	12	24.04851	24.0485x
78	12	1128322222222	112.832222222222
81	22	x48	x.48
89	3	12183+	.12183+
ib.		12184—	.12184—
90	28	3 *in the Dividend*	3 *in the Quotient*
92	37	(.041764	(.41764